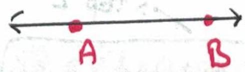
Math 2

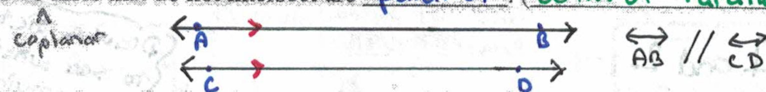
Unit 5.5 Postulates, Theorems, & Proofs

- **Postulate** – (aka an axiom) is a statement that is ACCEPTED as true without proof. These statements are often so obvious that there is no reason to prove them.

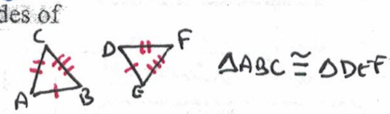
ie.. Through any two points is exactly one line. (Line Postulate)



Two lines that do not intersect are parallel. (Defn. of Parallel lines)

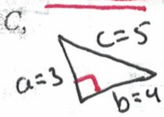


Three sides of a triangle are congruent to three sides of another triangle, then the triangles are congruent. (by the Side-Side-Side Postulate)



- **Theorem** – is a statement or a conjecture that has **been proven** or **is to be proven** true.

ie.. If a triangle is a right triangle with right angle *C*, then $a^2+b^2=c^2$. (Pythagorean Theorem)



$(3)^2+(4)^2=(5)^2$

$25=25$ ✓

If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Thm)

Both postulates and theorems are used to PROVE something in geometry. This is done by writing a PROOF.

A PROOF is a logical argument in which each statement you make is supported by a statement that is accepted as true. (Postulate, Theorem, Definition, Properties, or is Given).

Make a Statement, give a Reason.

The Proof Process

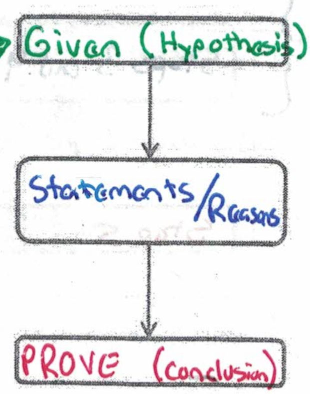
Step 1: List the given information and if possible, draw a diagram to illustrate this information.

Step 2: State the theorem or conjecture to be proven.

Step 3: Create a deductive argument by forming a logical chain of statements linking the given to what you are trying to prove.

Step 4: Justify each statement with a reason. Reasons include Definitions, Properties, Theorems, Postulates.

Step 5: State what it is that you have proven.(This why you did the proof)



There are several different types of proofs used in geometry:

- **Paragraph Proof** (aka *informal proof*)

  This type of proof involves writing a Paragraph to explain why a statement/conjecture for a given situation is true. It still follows the "*Proof Process*" and is just as valid as any other type of proof.

Example 1

Step 1 Step 2

Given that M is the midpoint of $\overline{XY}$ write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

Step 1 and 2

Given: M is the midpoint of $\overline{XY}$
Prove: $\overline{XM} \cong \overline{MY}$

X M Y

Draw a picture to see what you are proving!

Steps 3 and 4

If M is the midpoint of $\overline{XY}$, then from the definition of "midpoint of a segment", we know that $XM = MY$. This means that $\overline{XM}$ and $\overline{MY}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent.

Step 5

Thus, $\overline{XM} \cong \overline{MY}$.

- **Two Column Proof** (aka *formal proof and algebraic proof*)

  This type of proof involves using Statements and reasons organized in two columns.

Given: ~~~
Prove: ~~~

Statements	Reasons
1)	1) Given
2) - - - -	2) - - -
3) - - -	3) - - -
4) PROOF	4) - - - -

Example 2

Prove that if $-5(x + 4) = 70$, then $x = -18$.

Steps 1 and 2

Given: $-5(x + 4) = 70$
Prove: $x = -18$

This is what YOU HAVE to do!

Steps 3 and 4

Statements	Reasons
$-5(x + 4) = 70$	Given
$(-5)x + (-5)4 = 70$	Distributive Prop. of =
$-5x - 20 = 70$	Substitution Prop. of =
$-5x - 20 + 20 = 70 + 20$	Addition Prop of =
$-5x = 90$	Substitution Prop of =
$\frac{-5x}{-5} = \frac{90}{-5}$	Division Prop of =
$x = -18$	Substitution Prop. of =

Step 5

$-5(x+4) = 70$

$-5x - 20 = 70$
$+20 \quad +20$

$\frac{-5x}{5} = \frac{90}{5}$

$x = -18$

* Your Last Statement will always be what you are trying to prove!

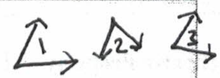
FOR EVERY STATEMENT, YOU MUST GIVE A REASON!

Example 3

Prove that $\angle 1 \cong \angle 2$, given $\angle 1$ and $\angle 3$ are complementary and $\angle 2$ and $\angle 3$ are complementary.

Steps 1 and 2

Given: $\angle 1$ and $\angle 3$ are complementary
$\angle 2$ and $\angle 3$ are complementary
Prove: $\angle 1 \cong \angle 2$



Steps 3 and 4

Statements	Reasons
$\angle 1$ and $\angle 3$ are complementary; $\angle 2$ and $\angle 3$ are complementary	Given
$m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	Defn of Complementary ∠'s
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	Substitution Prop. of =
$m\angle 3 = m\angle 3$	Reflexive Prop.
$m\angle 1 = m\angle 2$	Substitution Prop of =
$\angle 1 \cong \angle 2$	Defn of ≅ ∠'s.

Step 5

- **Flow Proof**

This type of proof uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box.

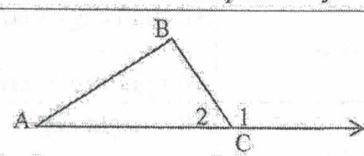
Example 4 Given $\triangle ABC$, prove $m\angle A + m\angle B = m\angle 1$

~~Prove that ∠1 ≅ ∠2, given ∠1 and ∠3 are complementary and ∠2 and ∠3 are complementary.~~

Steps 1 and 2

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B = m\angle 1$



Steps 3 and 4

$\boxed{\triangle ABC}$
Given

$\boxed{\angle 1 \text{ and } \angle 2 \text{ are Linear Pair}}$
Defn. of Linear Pair

$\boxed{\angle 1 \text{ and } \angle 2 \text{ are supplementary } \angle}$
If two ∠'s form a Linear pair = 180°

$\boxed{m\angle A + m\angle B + m\angle 2 = 180^\circ}$
Triangle Angle Sum Thm

$\boxed{m\angle 1 + m\angle 2 = 180^\circ}$
Defn of Supplementary ∠s

$\boxed{m\angle A + m\angle B + m\angle 2 = m\angle 1 + m\angle 2}$
Substitution Prop of =

Step 5

$\boxed{m\angle A + m\angle B = m\angle 1}$
Subtraction Prop of =