Math 2 Unit 5.4 Similarity Name: ____________

## Basic Idea of Polygon Similarity

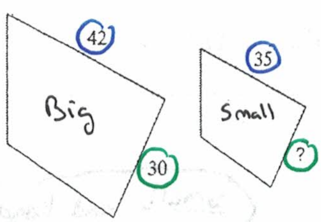
Similarity → Two <u>polygons are similar</u> if their corresponding parts as in their <u>angles are congruent</u> and their <u>sides are proportional.</u>

- The following symbol stands for SIMILARITY: <u>$\sim$</u>
- Similar polygons have the <u>same shape</u>, BUT <u>NOT the same size</u>
- If polygons are similar, they have a <u>scale factor</u> which is a <u>constant ratio</u> between the two figures' <u>corresponding sides</u>.

Scale factor is written as $\frac{\text{small \#}}{\text{big \#}}$ (ratio as a fraction) or <u>small # : large #</u> "to" (ratio as a statement)

**Example 1: The following set of polygons are similar, find what is asked.** READ CAREFULLY!

a.) Find missing side.



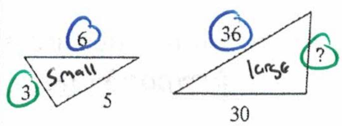

Set up a proportion

$\frac{35}{42} = \frac{x}{30}$

$\frac{42x}{42} = \frac{1050}{42}$

$x = 25$

b.) Find missing side.

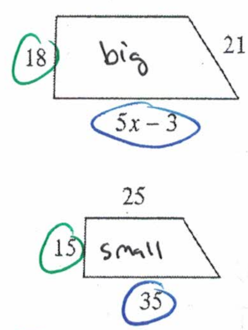



$\frac{6}{36} = \frac{3}{x}$

$6x = 108$

$x = 18$

c.) Find the value of x.




$\frac{35}{5x-3} = \frac{15}{18}$

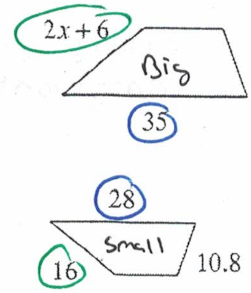
$15(5x-3) = 630$

$75x - 45 = 630$

$75x = 675$

$x = 9$

d.) Find missing length.




Step 1

$\frac{28}{35} = \frac{16}{2x+6}$

$28(2x+6) = 560$

$56x + 168 = 560$

$56x = 392$

$x = 7$

Step 2

$2x + 6$

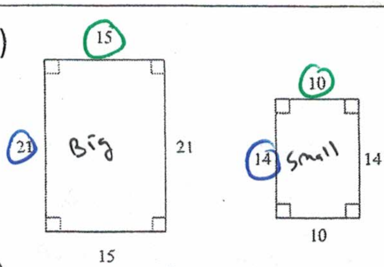
$2(7) + 6$

$14 + 6$

$20$

**Example 2: Determine if the two given figures are similar. If they are, state the scale factor.**

a.)



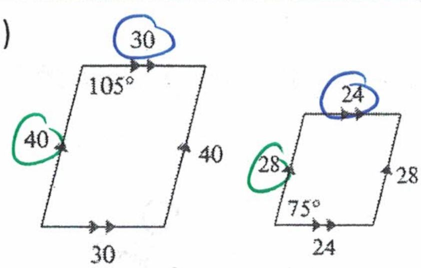

① $\frac{14}{21} = \frac{2}{3}$ ✓ ② $\frac{10}{15} = \frac{2}{3}$ ✓

Figures similar? Yes No

If yes, state scale factor: <u>2/3</u>

2:3

b.)

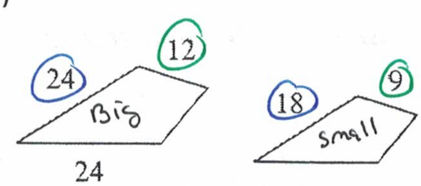



① $\frac{24}{30} = \frac{4}{5}$ ✗ ② $\frac{28}{40} = \frac{7}{10}$ ✗

Figures similar? Yes No

If yes, state scale factor: ———

c.)




① $\frac{9}{12} = \frac{3}{4}$ ✓ ② $\frac{18}{24} = \frac{3}{4}$ ✓

Figures similar? Yes No

If yes, state scale factor: <u>3/4</u>

# Triangle Similarity

**Similarity** → Two triangles are SIMILAR if the triangles' corresponding parts as in their angles are congruent and their sides are proportional.

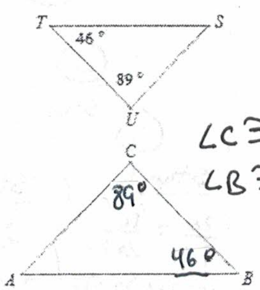
- The following symbol stands for SIMILARITY: $\sim$

ie: $\triangle ABC \sim \triangle A'B'C'$ read as "triangle ABC is **SIMILAR TO** triangle A'B'C' "

- Similar triangles have the Same shape, BUT NOT the same size.
- If triangles are similar, they have a scale factor which is a constant ratio between the two figures' corresponding sides.

## Properties of Similar Triangles

Corresponding angles are congruent.

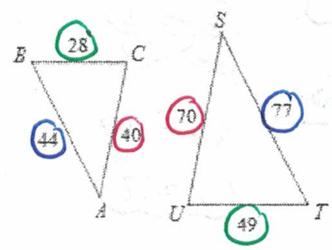



$\angle C \cong \angle U$
$\angle B \cong \angle T$

b/c of the "Sum of Interior Angles" of a triangle must equal 180°, the corresponding third angle must be congruent.

$\triangle ABC \sim \triangle STU$

Corresponding sides are in the same proportion.




small and keep big
all ratios the same!

$$\frac{AB}{ST} = \frac{AC}{SU} = \frac{BC}{TU}$$

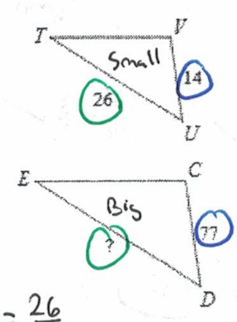
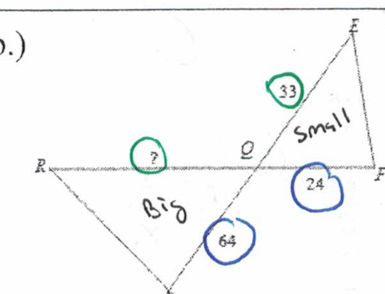
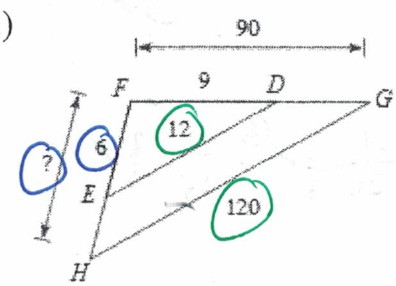
$\triangle ABC \sim \triangle STU$

$$\frac{44}{77} = \frac{40}{70} = \frac{28}{49}$$

$4/7$

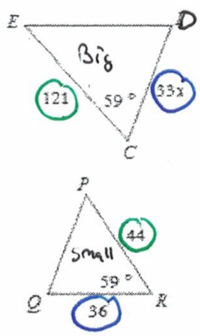
Scale factor $4/7$

**Example1: Each set of triangles are similar, find what is asked for.**

| a.) | b.) | c.) |
|---|---|---|
|  |  |  |
| $\frac{14}{77} = \frac{26}{x}$ | $\frac{24}{64} = \frac{33}{x}$ | $\frac{6}{x} = \frac{12}{120}$ |
| $14x = 2002$ | $24x = 2112$ | $12x = 720$ |
| $x = 143$ | $x = 88$ | $x = 60$ |

**Example2**: **Solve for x.**

**a.)** $\triangle CDE \sim \triangle RQP$

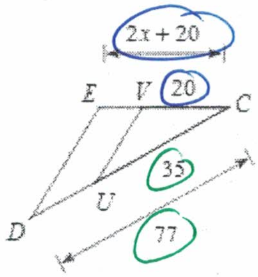



$\frac{36}{33x} = \frac{44}{121}$

$1452x = 4356$

$x = 3$

**b.)**




$\frac{20}{2x+20} = \frac{35}{77}$

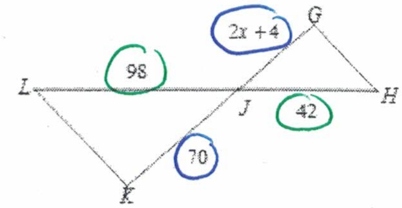
$35(2x+20) = 1540$

$70x + 700 = 1540$

$70x = 840$

$x = 12$

**c.)** $\triangle JKL \sim \triangle JGH$




$\frac{2x+4}{70} = \frac{42}{98}$

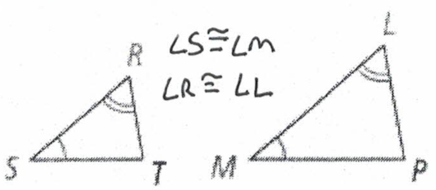
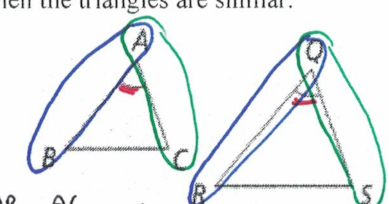
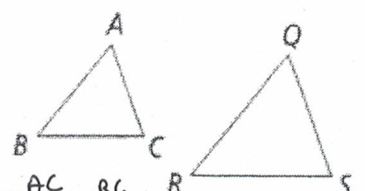
$98(2x+4) = 2940$

$196x + 392 = 2940$

$196x = 2548$

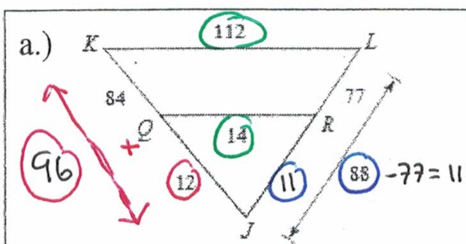
$x = 13$

Any triangle is defined by 6 measures: 3 sides and 3 angles. However, you do not need to know all six measures. The following groups of 3 will do:

| Important Triangle Similarity Postulates and Theorems | | |
|---|---|---|
| **AA ~ Similarity Postulate** | **SAS ~ Similarity Theorem** | **SSS ~ Similarity Theorem** |
| If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. | If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar. | If the corresponding sides of two triangles are proportional, then the triangles are similar. |
|   |   |   |
| $\angle S \cong \angle M$<br>$\angle R \cong \angle L$<br>$\triangle SRT \sim \triangle MLP$ by AA~ Post. | $\frac{AB}{QR} = \frac{AC}{QS}$ and $\angle A \cong \angle Q$<br>$\triangle ABC \sim \triangle QRS$ by SAS ~ Thm | $\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$<br>$\triangle ABC \sim \triangle QRS$ by SSS ~ Thm |
| (The third angle has to be congruent!) | (Two pairs of sides in the same proportion and the included angle equal.) | (All three pairs of corresponding sides are in the same proportion.) |

**Example3**: **State if the triangles in each pair are similar. If so, state how you know they are similar and complete the similarity statement.**

**a.)**

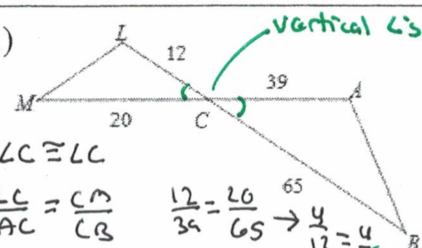



$\triangle JKL \sim \triangle JQR$ by SSS~ Thm

$\frac{12}{96} = \frac{14}{112} = \frac{11}{88}$

$\frac{1}{8} = \frac{1}{8} = \frac{1}{8}$

**b.)**



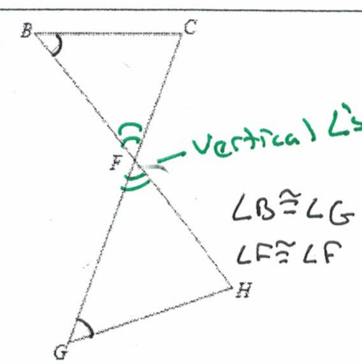

$\angle C \cong \angle C$

$\frac{LC}{AC} = \frac{CM}{CB}$

$\frac{12}{39} = \frac{20}{65} \rightarrow \frac{4}{13} = \frac{4}{13}$

$\triangle CBA \sim \triangle CML$ by SAS ~ Thm

**c.)**




$\angle B \cong \angle G$

$\angle F \cong \angle F$

$\triangle FGH \sim \triangle FBC$ by AA~ Post.