

## 4.2 - Solving Systems of Equations By Substitution

This eqn is solved for "y"

### Solving Systems By Substitution Method Steps

$$\begin{cases} y = 2x + 5 \\ 2x - 7y = 10 \end{cases}$$

- 1.) This method is IDEAL for equations that are ALREADY SOLVED for a variable; it DOES NOT MATTER which variable is solved for.
- 2.) If an equation is Not solved for a variable then choose the EASIEST equation to solve for.  
 Ex:  $\begin{cases} x + 2y = 7 \\ 3x - 5y = 12 \end{cases}$  ← This eqn is the easiest to solve for a variable; x!
- 3.) Use the expression from the solved variable and SUBSTITUTE it into the other equation.
- 4.) Should have an equation with only ONE variable now! → solve for the remaining variable
- 5.) Once found the value of one variable, then substitute that value into the expression in step 2.
- 6.) Your solution (answer) should be an Ordered Pair but you may have some special cases....
  - One Solution →  $(x, y)$ ; the variables DO NOT cancel out!  
 consistent independent ie:  $5 = 5$  ✓
  - Infinitely Many Solution → IMS; the variables CANCEL OUT and you are left with a TRUE statement.  
 consistent dependent (same lines)
  - No Solution →  $\emptyset$ ; the variables CANCEL OUT and you are left with a FALSE statement.  
 Inconsistent (Parallel lines) ie:  $1 \neq 2$

**Example 1:** Solve each system using the substitution method.

<p>a.) <math>\begin{cases} y = -4x - 8 \\ y = x + 7 \end{cases}</math></p> <p>* This is the <u>IDEAL</u> system to use <u>SUBSTITUTION</u> on. B/c both eqns are solved for a variable.</p> <p><math>y = -4x - 8</math>  <math>(x + 7) = -4x - 8</math>  <math>x + 7 = -4x - 8</math>  <math>+4x \quad +4x</math>  <math>5x + 7 = -8</math>  <math>-7 \quad -7</math>  <math>5x = -15</math>  <math>\frac{5x}{5} = \frac{-15}{5}</math>  <math>x = -3</math></p> <p><math>y = x + 7</math>  <math>y = (-3) + 7</math>  <math>y = 4</math></p> <p><math>(-3, 4)</math></p> <p>This is only half of your solution!</p>	<p>b.) <math>\begin{cases} y = 2x \\ 6x - y = -8 \end{cases}</math></p> <p><math>6x - y = -8</math>  <math>6x - (2x) = -8</math>  <math>6x - 2x = -8</math>  <math>4x = -8</math>  <math>\frac{4x}{4} = \frac{-8}{4}</math>  <math>x = -2</math></p> <p><math>y = 2x</math>  <math>y = 2(-2)</math>  <math>y = -4</math></p> <p><math>(-2, -4)</math></p> <p>only half of solution.</p> <p>What do these solutions represent?              This is where the two lines intersect.</p>	<p>c.) <math>\begin{cases} y - 4 = 0.5x \\ x - 2y = -8 \end{cases}</math></p> <p>Solve for x → <math>x - 2y = -8</math>  <math>+2y \quad +2y</math>  <math>x = 2y - 8</math></p> <p><math>y - 4 = 0.5x</math>  <math>y - 4 = 0.5(2y - 8)</math>  <math>y - 4 = y - 4</math> ← Both sides are EXACTLY the same.  <math>-y \quad -y</math>  <math>-4 = -4</math> ← True statement and no variables.</p> <p><b>IMS</b></p> <p>What does this solution represent?              The two lines are the same!</p>
--	---	--

<p>d.) <math>\begin{cases} 2x + 3y = 4 \\ y - 6x = -7 \end{cases}</math></p>	<p>e.) <math>\begin{cases} y = 3x + 1 \\ 6x - 2y = 10 \end{cases}</math></p>	<p>f.) <math>\begin{cases} 2x + 8y = -8 \\ 3x + 5y = 2 \end{cases}</math></p>
<p><math>y = 6x - 7</math></p> <p><math>2x + 3(6x - 7) = 4</math></p> <p><math>2x + 18x - 21 = 4</math></p> <p><math>20x - 21 = 4</math></p> <p><math>20x = 25</math></p> <p><math>x = 1.25</math></p> <p><math>y = 6(1.25) - 7</math></p> <p><math>y = 7.5 - 7</math></p> <p><math>y = .5</math></p> <p><math>(1.25, .5)</math></p>	<p><math>6x - 2y = 10</math></p> <p><math>6x - 2(3x + 1) = 10</math></p> <p><math>6x - 6x - 2 = 10</math></p> <p><math>-2 = 10</math></p> <p>Variables cancel out and a FALSE statement remains!</p> <p><math>\emptyset</math></p> <p>What does this solution represent? The lines do not intersect, they are parallel.</p>	<p><math>2x + 8y = -8</math></p> <p><math>2x + 8y = -8</math></p> <p><math>2x = -8y - 8</math></p> <p><math>x = -4y - 4</math></p> <p><math>3x + 5y = 2</math></p> <p><math>3(-4y - 4) + 5y = 2</math></p> <p><math>-12y - 12 + 5y = 2</math></p> <p><math>-7y - 12 = 2</math></p> <p><math>-7y = 14</math></p> <p><math>y = -2</math></p> <p><math>x = -4(-2) - 4</math></p> <p><math>x = 8 - 4</math></p> <p><math>x = 4</math></p> <p><math>(4, -2)</math></p>

**Example 2: Complete each problem. Make sure to find what the problem requests!**

a.) Your school is planning an after-school trip for 193 students. There are eight drivers available and two types of vehicles school buses and minivans. The school buses seat 51 people each and the minivans seat 8 people. How many buses and minivans will be needed? *Systems of Eqns: 2 variables, 2 eqns!*

b = # of buses needed  
m = # of minivans

eqn 1:  $193 = 51b + 8m$

eqn 2:  $8 = b + m$

$m = 8 - b$

$193 = 51b + 8(8 - b)$

$193 = 51b + 64 - 8b$

$193 = 43b + 64$

$129 = 43b$

$b = 3$

$8 = b + m$

$8 = 3 + m$

$m = 5$

The school needs 3 buses and 5 minivans.

b.) There are 1170 students in a school. The ratio of girls to boys is 23 : 22. How many more girls are there than boys?

b = # of boys  
g = # of girls

eqn 1:  $1170 = b + g \rightarrow b = 1170 - g$

eqn 2:  $\frac{23}{22} = \frac{g}{b}$

$23(1170 - g) = 22g$

$26,910 - 23g = 22g$

$26,910 = 45g$

$g = 598$

$b = 572 < b = 1170 - g$

$598 - 572 = 26$   
There are 26 more girls than boys.

c.) Find the sum of x and y given:  
 $y = 9x + 3480$  and  $y = 81x - 7104$ .

*\*Find the ordered pair first!\**

$y = 9x + 3480$

$y = 81x - 7104$

$9x + 3480 = 81x - 7104$

$-72x + 3480 = -7104$

$-72x = -10584$

$x = 147$

$y = 9x + 3480$

$y = 9(147) + 3480$

$y = 1323 + 3480$

$y = 4803$

$147 + 4803 = 4950$

The sum of x and y is 4950.