4.1 – Solving Systems of Equations By Graphing

Solving Systems By Graphing

*- **system of (linear) equations** → is TWO or MORE linear equations grouped together.

Ex: $y = 2x + 5$, $3x - 4y = 20$ → $\begin{cases} y = 2x+5 \\ 3x-4y=20 \end{cases}$

- There are three ways to solve a system of linear equations:

1.) by GRAPHING (Hand / ~~Calculator~~) 2.) by SUBSTITUTION Method 3.) by ELIMINATION method

* Solutions of Systems of Linear Equations

a.) By Graphing (method) (VISUAL) → is the common point shared or the point of intersection (x, y)

b.) By Substitution/Elimination (methods) (Algebra) → is the ordered pair (x, y) that makes all equations true

Example 1: Determine if the ordered pair (– 1 , 5) is a solution to each system. State YES or NO.

MUST SHOW WORK! Substitute the ordered pair into each eqn of the system.

a.) $y = 2x + 7$, $y = x + 6$ (-1, 5) x y

$y = 2x+7$: $(5) = 2(-1) + 7$; $5 = -2 + 7$; $5 = 5$ ✓

$y = x + 6$: $(5) = (-1) + 6$; $5 = -1 + 6$; $5 = 5$ ✓

Yes, b/c the ordered pair makes all eqns true.

b.) $y = -x + 4$, $5x - y = 0$ (-1, 5)

$y = -x + 4$: $(5) = -(-1) + 4$; $5 = 1 + 4$; $5 = 5$ ✓

$5x - y = 0$: $5(-1) - (5) = 0$; $-5 - 5 = 0$; $-10 = 0$ ✗

No, b/c the ordered pair does not make all eqns. true

c.) $2x + 3y = 13$, $3x - y = -8$

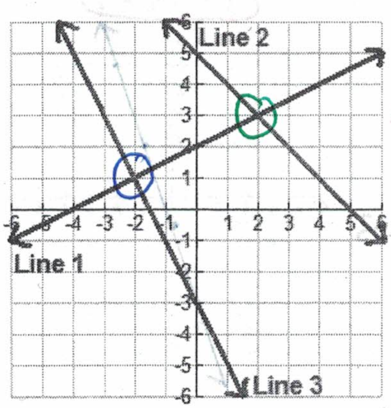
$2x + 3y = 13$: $2(-1) + 3(5) = 13$; $-2 + 15 = 13$; $13 = 13$ ✓

$3x - y = -8$: $3(-1) - (5) = -8$; $-3 - 5 = -8$; $-8 = -8$ ✓

Yes b/c the ordered pair makes all eqns. true.

Example 2: Determine the solution to each system by finding the point of intersection.

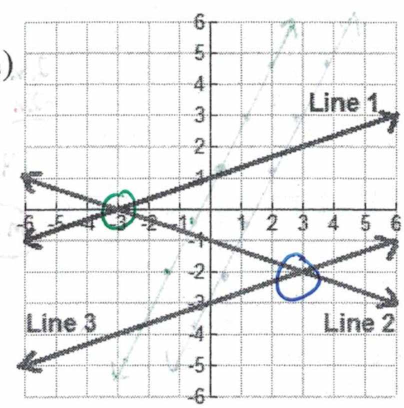
a.)



Solution (L1 / L2): (2, 3)

Solution (L1 / L3): (-2, 1)

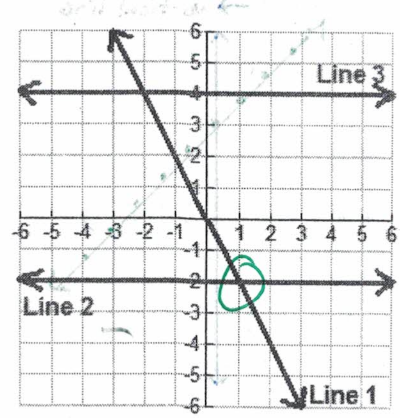
b.)



Solution (L1 / L2): (-3, 0)

Solution (L2 / L3): (3, -2)

c.)



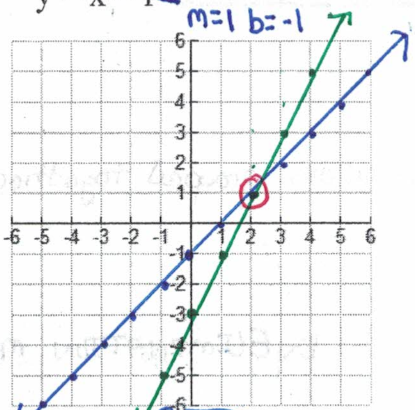
Solution (L1 / L2): (1, -2)

Solution (L2 / L3): No Solution (Parallel lines do not intersect.)

Example 3: Solve each system by graphing each line and determine the point of intersection.

a.) $y = 2x - 3$ — $m=2$ $b=-3$

$y = x - 1$ — $m=1$ $b=-1$



Solution : $(2,1)$

b.) $y = -\frac{1}{2}x + 2$ — $m=-\frac{1}{2}$ $b=2$

$3x + 4y = 12$

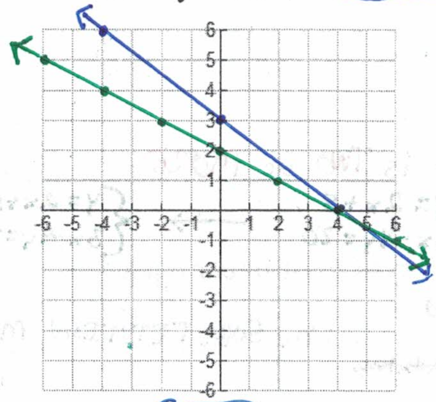
Transform to $y=mx+b$

$3x+4y=12$

$-3x \qquad -3x$

$\frac{4y}{4}=\frac{-3x}{4}+\frac{12}{4}$

$y=-\frac{3}{4}x+3$; $m=-\frac{3}{4}$ $b=3$



Solution: $(4,0)$

Classifying and Analyzing Special Types of Systems (of Equations)

- There are three types of systems of linear equations and each can be classified based on their solution:

1.) No Solution $\varnothing$ → when given Parallel Lines (same slope) that Do NOT intersect.
but different y-intercepts.
this is classified as an Inconsistent system of equations.

2.) Infinitely Many Solutions IMS → when given the same lines where the lines overlap each other
this is classified as a Consistent Dependent system of equations.

3.) One solution (x,y) → when given different lines that intersect each other at one point
this is classified as a Consistent Independent system of equations.

$6x-2y=2$

$-6x \qquad -6x$

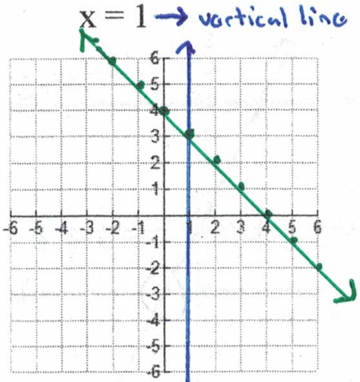
$\frac{-2y}{-2}=\frac{-6x}{-2}+\frac{2}{-2}$

$y=3x-1$

Example 4: Graph each system, classify it, and determine the type of solution.

a.) $y = -x + 4$ — $m=-1$, $b=4$

$x = 1$ → vertical line



Type of Solution: One solution

Classification: Consistent Independent

Solution: $(1,3)$

b.) $y = -2x + 1$ — $m=-2$, $b=1$

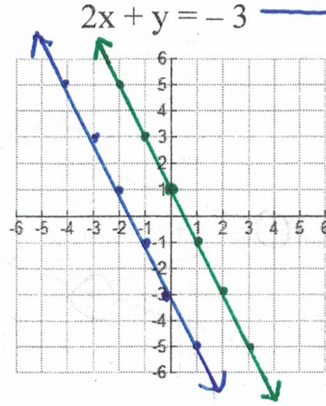
$2x + y = -3$

$2x+y=-3$

$-2x \qquad -2x$

$y=-2x-3$

$m=-2$ $b=-3$



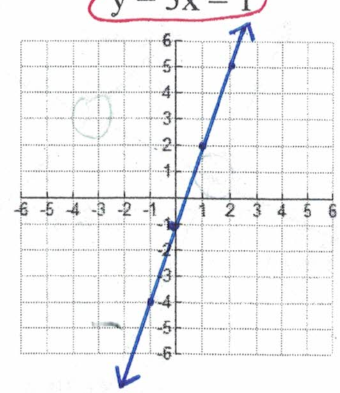
Type of Solution: No Solution

Classification: Inconsistent

Solution: $\varnothing$

c.) $6x - 2y = 2$

$y = 3x - 1$



Type of Solution: IMS

Classification: Consistent Dependent

Solution: IMS