Unit 5.3 – Triangle Congruency Part I: SSS and SAS Triangles

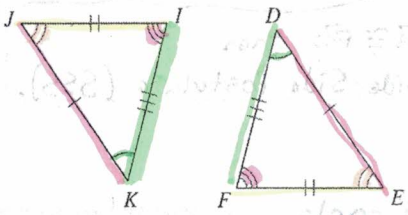
Basic Idea of Triangle Congruency

Congruence → Two triangles are congruent if their corresponding parts as in their sides and angles have the SAME MEASUREMENTS!.

- The following symbol stands for CONGRUENCE: $\cong$
- When naming congruent polygons, you must keep the letters in ORDER of Congruence which will help you to tell very specific congruent parts.
- Reflexive Property – If two figures SHARE a side, then that side is congruent in both figures
- Vertical Angles – If two angles are formed by opposite rays, then both angles are congruent

Example 1: List the corresponding parts that are congruent, then write a congruent statement.

a.)

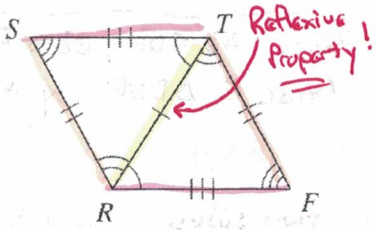


Sides: $\overline{JK} \cong \overline{ED}$, $\overline{JI} \cong \overline{EF}$, $\overline{KI} \cong \overline{DF}$

Angles: $\angle K \cong \angle D$, $\angle J \cong \angle E$, $\angle I \cong \angle F$

Congruence Statement: $\triangle JKI \cong \triangle EDF$

b.)

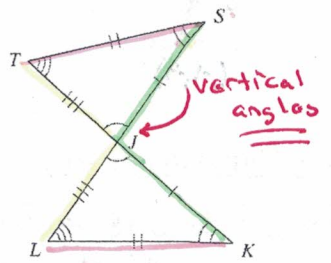


Sides: $\overline{RT} \cong \overline{TR}$, $\overline{SR} \cong \overline{FT}$, $\overline{ST} \cong \overline{FR}$

Angles: $\angle S \cong \angle F$, $\angle RTS \cong \angle TRF$, $\angle TRS \cong \angle RTF$

Congruence Statement: $\triangle TRS \cong \triangle RTF$

c.)



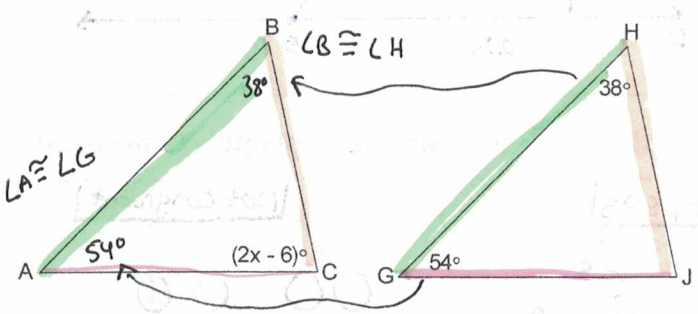
Sides: $\overline{JS} \cong \overline{JK}$, $\overline{ST} \cong \overline{KL}$, $\overline{TJ} \cong \overline{LJ}$

Angles: $\angle J \cong \angle J$, $\angle S \cong \angle K$, $\angle T \cong \angle L$

Congruence Statement: $\triangle JST \cong \triangle JKL$

Example 2: Given a congruence statement, find what is asked.

a.) If $\triangle ABC \cong \triangle GHJ$, then find the value of x.



$\angle B \cong \angle H$

$\angle A \cong \angle G$

$54 + 38 + 2x - 6 = 180$

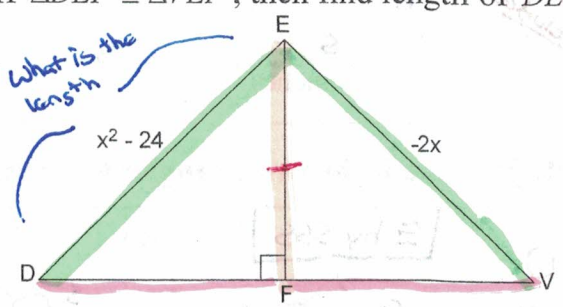
$92 + 2x - 6 = 180$

$86 + 2x = 180$

$2x = 94$

$x = 47$

b.) If $\triangle DEF \cong \triangle VEF$, then find length of $\overline{DE}$



$x^2 - 24 = -2x$

$x^2 + 2x - 24 = 0$

Solve the Quadratic!

$(x + 6)(x - 4) = 0$

$x = -6 \quad x = 4$

✓ to see which one works!

$\overline{DE} = x^2 - 24$

$= (-6)^2 - 24$

$= 36 - 24$

$\overline{DE} = 12$ units

$\overline{DE} = x^2 - 24$

$= (4)^2 - 24$

$= 16 - 24$

$\overline{DE} = -8$

Length cannot be negative

$DE = 12$ units

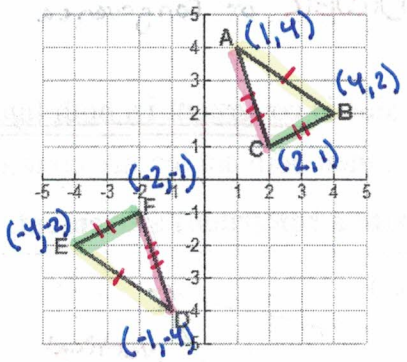
Triangle Congruence by Side-Side-Side (SSS)

Side-Side-Side (SSS) Postulate → If three sides of one triangle are congruent to three sides in another triangle, then the triangles must be congruent.

** Note: To prove the SSS postulate you need to remember the Distance Formula →

If have (x_1, y_1) and (x_2, y_2), then distance between the two points is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Proving the SSS Postulate using Algebra – Prove that ΔABC is congruent to ΔDEF



$\overline{AB} = \sqrt{(1-4)^2 + (4-2)^2} = \sqrt{13} \approx 3.6$ $\quad$ $\overline{DE} = \sqrt{(-1--4)^2 + (-4-2)^2} = \sqrt{13} \approx 3.6$

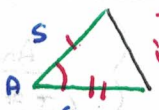
$\overline{BC} = \sqrt{(4-2)^2 + (2-1)^2} = \sqrt{5} \approx 2.2$ $\quad$ $\overline{EF} = \sqrt{(-4--2)^2 + (-2--1)^2} = \sqrt{5} \approx 2.2$

$\overline{CA} = \sqrt{(2-1)^2 + (1-4)^2} = \sqrt{10} \approx 3.2$ $\quad$ $\overline{FD} = \sqrt{(-2--1)^2 + (-1-4)^2} = \sqrt{10} \approx 3.2$

Since $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$ then $\triangle ABC \cong \triangle DEF$ by the Side-Side-Side Postulate (SSS).

Triangle Congruence by Side-Angle-Side (SAS)

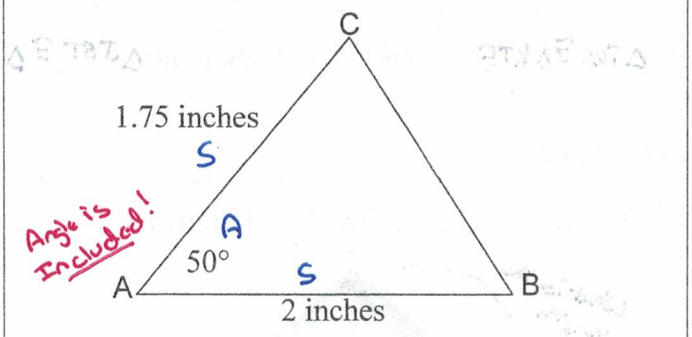
Side-Angle-Side (SAS) Postulate → If two sides and the Included angle of one triangle are congruent to two sides and the included angle in another triangle, then the triangles must be congruent.

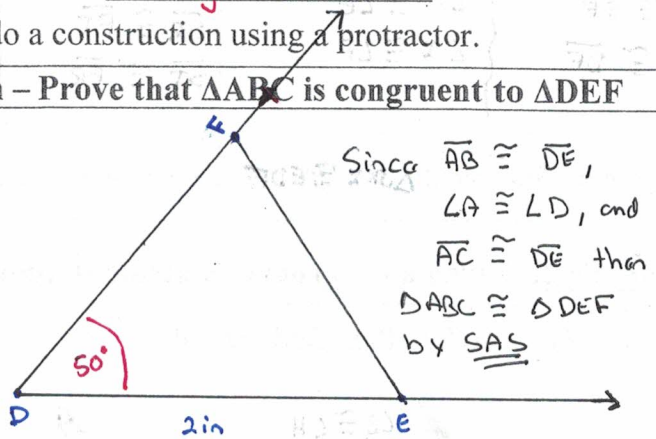


The angle is in between the 2 sides!

** Note: To prove the SAS postulate you will need to do a construction using a protractor.

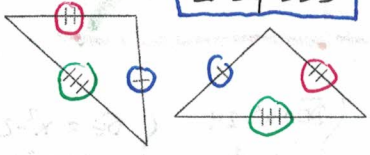
Proving the SAS Postulate using a Construction – Prove that ΔABC is congruent to ΔDEF

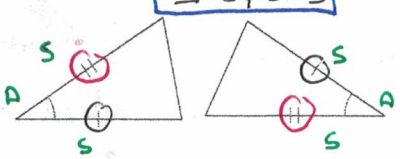


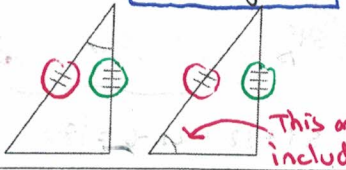


Since $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$ then $\triangle ABC \cong \triangle DEF$ by SAS

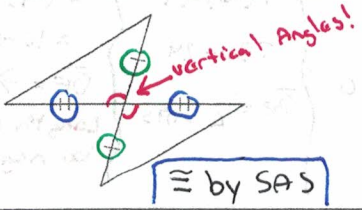
Example 3: State if the two triangles are congruent. If they are, state by the appropriate postulate.

a.)  ≅ by SSS

b.)  ≅ by SAS

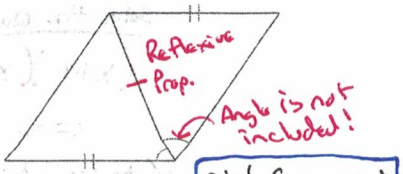
c.)  Not Congruent

This angle is not included.

d.) 

vertical Angles!

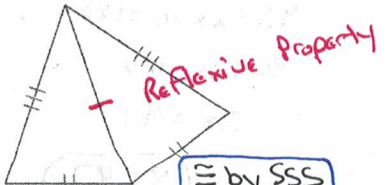
≅ by SAS

e.) 

Reflexive Prop.

Angle is not included!

Not Congruent

f.) 

Reflexive Property

≅ by SSS