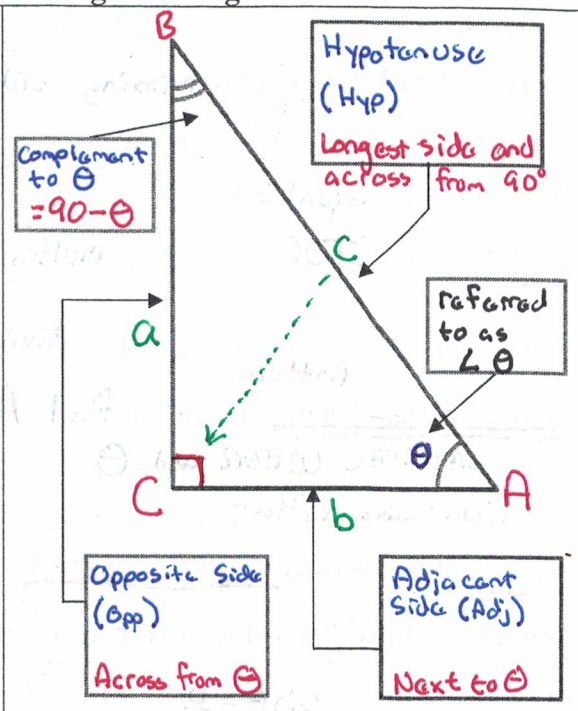
6.2 – Trigonometric Ratios in Right Triangles

The Three Basic Trigonometric Ratios in Right Triangles

Word	Abbreviation	Definition
Sine	Sin(θ)	$\sin\theta = \frac{Opp}{Hyp}$
Cosine	Cos(θ)	$\cos\theta = \frac{Adj}{Hyp}$
Tangent	tan(θ)	$\tan\theta = \frac{opp}{Adj}$



Notes: 1.) The symbol θ (called Theta) is a Greek Letter that is often used to refer or indicate Angles.

2.) You may have to use Pythagorean Thm to find third side.

– **rationalizing (the denominator)** → way to simplify an expression with radicals (square roots) by NOT ALLOWING a radical in the denominator (we will have do this in Example 2) so let's review/practice this concept…

Example 1: Let's review on how to "Rationalize the Denominator".

a.) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ → $\frac{4\sqrt{5}}{5}$

b.) $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ → $\frac{6\sqrt{3}}{3} \rightarrow 2\sqrt{3}$

c.) $\frac{21}{24\sqrt{2}}$ → $\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ → $\frac{\sqrt{2}}{2\sqrt{4}} \rightarrow \frac{\sqrt{2}}{4}$

d.) $\frac{48}{3\sqrt{6}}$ → $\frac{16}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$ → $\frac{16\sqrt{6}}{6} \rightarrow \frac{8\sqrt{6}}{3}$

Example 2: Find the value of the three basic trigonometric ratios.

a.) 8 (Adj), θ, 6 (Opp), 10 Hyp

$(8)^2 + (6)^2 = (H)^2$, $100 = H^2$, $H = 10$

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

$\frac{6}{10}$ $\frac{8}{10}$ $\frac{6}{8}$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

b.) $8\sqrt{2}$ (Adj), θ, (Opp) 4, (Hyp) 12

$(8\sqrt{2})^2 + (opp)^2 = (12)^2$

$128 + (Opp)^2 = 144$

$(Opp)^2 = 16$

$Opp = 4$

$\sin\theta = \frac{4}{12}$ $\cos\theta = \frac{8\sqrt{2}}{12}$ $\tan\theta = \frac{4}{8\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin\theta = \frac{1}{3}$ $\cos\theta = \frac{2\sqrt{2}}{3}$ $\tan\theta = \frac{4\sqrt{2}}{16}$

$\tan\theta = \frac{\sqrt{2}}{4}$

c.) Given $\tan\theta = \frac{\sqrt{3}}{3}$, find the other two trigonometric ratios.

$\tan\theta = \frac{O}{A}$ $Opp = \sqrt{3}$ $Adj = 3$

Find Hyp:

$(\sqrt{3})^2 + (3)^2 = Hyp^2$

$3 + 9 = Hyp^2$

$12 = H^2$

$2\sqrt{3} = Hyp$

$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}}$

$\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\cos\theta = \frac{3\sqrt{3}}{6}$

$\cos\theta = \frac{\sqrt{3}}{2}$

* Simplify your FRACTIONS *

* Reciprocals of 3 Basic Trig Functions *

sin → cosecant $\csc\theta = \frac{Hyp}{opp}$

cos → secant $\sec\theta = \frac{Hyp}{Adj}$

tan → cotangent $\cot\theta = \frac{Adj}{opp}$

* Check Your Calculator Mode - It needs to be in DEGREES! *

Finding a Missing Side with a Trigonometric Ratio and Solving a Right Triangle

- With trigonometry, you only need to know one side and one angle (other than the right angle) in order to find the missing side "x" in a right triangle.
- Decide how the given side and given angle relates to the missing side "x".
- Set up a(n) equation using the appropriate basic trig ratio (sin / cos / tan)
  - If "x" is on TOP, then multiply → Ex: $\tan 56° = \frac{x}{21}$ (21) → $21 \tan 56 = x$ $x = 31.1$
  - If "x" is on Bottom, then divide → Ex: $\sin 72° = \frac{16}{x}$ → $x = \frac{16}{\sin 72}$ $x = 16.8$
- Solving a right triangle means to find ALL missing sides and angles → will have 3 Answers!
  - CAPITAL LETTER and Θ always represent angles
  - lower case letter always represent side lengths (which are across from its angle)
- Round all answers to NEAREST TENTH! NO RADICALS in these sets of problems!

Example 3: Find the value of side x. Round to tenth place.

a.) $\sin\theta = \frac{O}{H}$ $\sin 54 = \frac{x}{6}$ $x = 6 \sin 54$ $x = 4.9$

b.) $\cos\theta = \frac{A}{H}$ $\cos 47.5 = \frac{7.4}{x}$ $x = \frac{7.4}{\cos 47.5}$ $x = 11$

c.) $\tan\theta = \frac{O}{A}$ $\tan 32 = \frac{12}{x}$ $x = \frac{12}{\tan 32}$ $x = 19.2$

Example 4: Solve triangle ABC. Round to tenth place.

a.) A = 33° and b = 5.8

① $\angle B = 90 - 33$ $\angle B = 57°$

② $\cos 33 = \frac{5.8}{H}$ $H = \frac{5.8}{\cos 33}$ $H = 6.9$

③ $\tan 33 = \frac{opp}{5.8}$ $opp = 5.8 \tan 33$ $opp = 3.8$

b.) B = 68° and c = 14

① $\angle A = 90° - 68°$ $\angle A = 22°$

② $\cos 68 = \frac{a}{14}$ $a = 14 \cos 68$ $a = 5.2$

③ $\sin 68 = \frac{b}{14}$ $b = 14 \sin 68$ $b = 13$

Example 5 – Critical Thinking: Find the length of side x. Round to tenth place.

a.)

① Find y: $\tan 38 = \frac{24}{y}$ $y = \frac{24}{\tan 38}$ $y = 30.7$

② Find z: $\tan 56 = \frac{24}{z}$ $z = \frac{24}{\tan 56}$ $z = 16.2$

③ $x = z + y$ $x = 16.2 + 30.7$ $x = 46.9$

b.)

① Find y: $y^2 + (32)^2 = (51)^2$ $y^2 = 1577$ $y \approx 39.7$

② Find z: $\tan 23 = \frac{32}{z}$ $z = \frac{32}{\tan 23}$ $z = 78.4$

③ $x = z - y$ $x = 75.4 - 39.7$ $x = 35.7$