5.2 Triangles

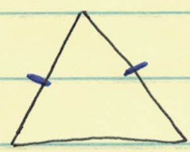
Triangles are categorized by ANGLE MEASURES and SIDES.
(named)

FYI
means congruent

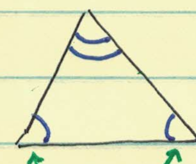
Angles			Sides
$\angle = 90°$	right triangle		Scalene $\triangle$ if 0 sides are $\cong$
$\angle < 90°$	acute triangle (All angles)		Isosceles $\triangle$ if 2 sides are $\cong$
$\angle > 90°$	obtuse triangle		Equilateral $\triangle$ if 3 sides are $\cong$

$\cong$ "congruent" m$\angle$ "measure of the angle"

$\triangle$ "triangle" ⊿ "right triangle"



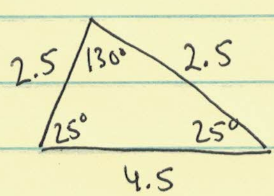
Two sides are $\cong$



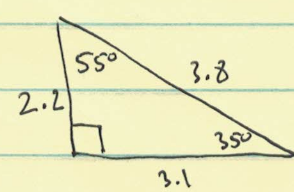
*Angle marks

Two angles are congruent

Examples



Obtuse Isosceles $\triangle$
(Angle) (sides)



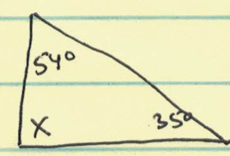
Right Scalene $\triangle$

There are four Basic Properties of Δs

A) Sum of Interior Angles

The sum of the measures of the interior angles of a Δ is $180°$. If you have two angles, you can find the measure of the third angle.

Example:



$x + 54° + 35° = 180°$

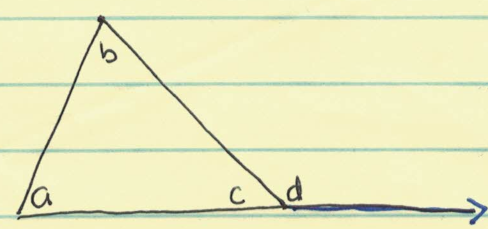
$x + 89 = 180$

$x = 91°$ * don't forget your units!

B) Measure of Exterior Angle

This angle is formed by extending one of the sides of a triangle. An exterior angle is ALWAYS SUPPLEMENTARY (sum = 180) with the interior angle with which it shares a vertex.

Example

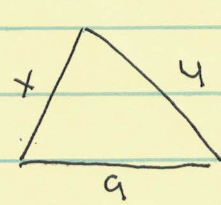


$c° + d° = 180°$

C) Triangle Inequality

The length of a side of a triangle is LESS THAN the **sum** of the other two sides AND is GREATER THAN the difference of the other two sides.

Example



$x < 9 + 4$ $\quad$ $x > 9 - 4$

$x < 13$ $\quad$ $x > 5$

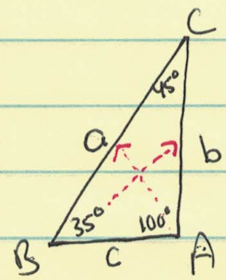
Side x: $5 < x < 13 \rightarrow (5, 13)$

D) Proportionality of Triangles

In **EVERY** triangle, the LONGEST SIDE is opposite the LARGEST ANGLE! The shortest side is opposite the smallest angle!

Example

Figure not drawn to scale!



* Angles are named using Capital letters

Sides are named using lower case letters.

"a" is largest side b/c $\angle A$ is largest angle.

"b" is smallest side b/c $\angle B$ is smallest angle.

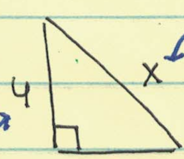
Pythagorean Theorem (only for right triangles)

$a^2 + b^2 = c^2$; where "a" and "b" are legs and "c" is the hypotenuse of a right triangle.

$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

always the longest side! It is across from the $90°$ angle.

Example:



hypotenuse

Legs

$a^2 + b^2 = c^2$

$(4)^2 + (3)^2 = c^2$

$16 + 9 = c^2$

$25 = c^2 \rightarrow \sqrt{25}$

$c = 5$

Are these the sides of a right triangle?

13, 5, 12

c b/c largest side!

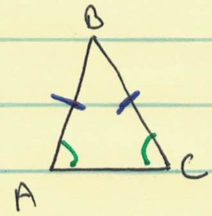
<u>Yes!</u>

$(5)^2 + (12)^2 \stackrel{?}{=} (13)^2$

$25 + 144 = 169$

$169 = 169$ ✓

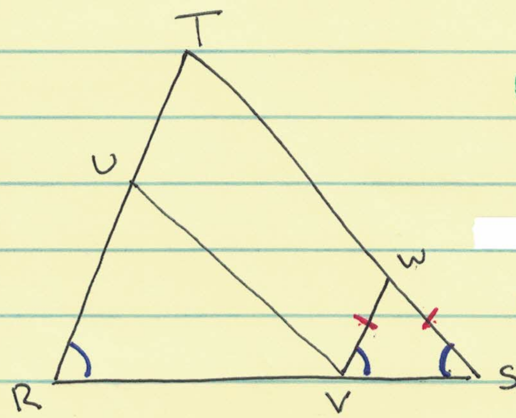
Isosceles Triangle Theorem



If two sides of a triangle are $\cong$, then the ANGLES opposite those sides are $\cong$.

If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

Example



Prove that $\overline{RT} \cong \overline{ST}$. (mini proof!)

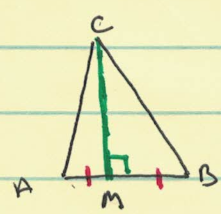
Statement	Reason
, $\overline{VW} \cong \overline{SW}$	Given
$\angle WVS \cong \angle WSV$	Given
$\angle S \cong \angle R$	Given
$\overline{RT} \cong \overline{ST}$	By the Isosceles Δ Theorem.

Perpendicular Bisector Theorem

means perpendicular

If a point is on a $\perp$ bisector of a segment, then it is EQUIDISTANT from the endpoints.

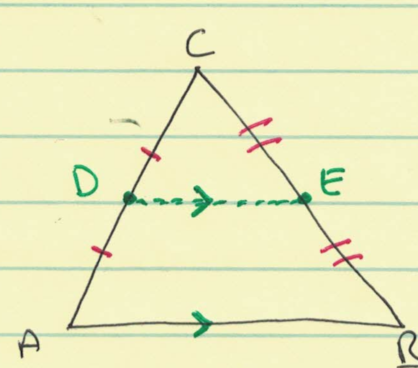
Example



If $\overline{CM} \perp \overline{AB}$, then $\overline{AM} \cong$ to $\overline{BM}$ and $\overline{AC} \cong \overline{BC}$.

Triangle Mid-Segment Theorem

If D is the mid-point of $\overline{AC}$ and E is the mid-point of $\overline{BC}$, then $\overline{AB} \parallel \overline{DE}$ and $\overline{DE} = \frac{1}{2}\overline{AB}$.



$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

only use if on a coordinate plane