6.1 – Review of Simplifying Square Roots and Pythagorean Theorem

Review of Simplifying Radicals: Square Roots

Index #

square root (radical)→ expression that contains a $\sqrt{\text{Radicand}}$ or $\sqrt[2]{\text{Radicand}}$ where the index # is 2

- GOAL to simplifying radicals – Factor Out "Prime #s" and group according to the index #

where the easiest way to do this is by breaking apart the Radicand using a factor tree

Prime ^ composite
Prime Prime

Example 1: Simplify each square root completely.
Left side – Show factor tree and Right side – Show work to simplify square root.

a.) Simplify: $\sqrt{64}$

$\sqrt{64}$ → 2, 32 → 2, 16 → 2, 8 → 2, 4 → 2, 2

2 2 2 2

Left is Prime!

End tree w/Prime #s

$\sqrt{64} = 2 \cdot 2 \cdot 2$

8

b.) Simplify: $\sqrt{27}$

$\sqrt{27}$ → 3, 9 → 3, $\boxed{3}$

3

What is not grouped goes back under the $\sqrt{\ }$.

$\sqrt{27} = 3 \cdot \sqrt{3}$

$3\sqrt{3}$

c.) Simplify: $\sqrt{180}$

$\sqrt{180}$ → 2, 90 → 2, 45 → 3, 15 → 3, $\boxed{5}$

2 3

$\sqrt{180} = 2 \cdot 3 \cdot \sqrt{5}$

$6\sqrt{5}$

d.) Simplify: $5\sqrt{28}$

$\sqrt{28}$ → 2, 14 → 2, $\boxed{7}$

2

$5 \cdot 2 \cdot \sqrt{7}$

$10\sqrt{7}$

e.) Simplify: $2\sqrt{24}$

$\sqrt{24}$ → 2, 12 → 2, 6 → $\boxed{2}\boxed{3}$

2

Both #'s go back in and you multiply them!

$2 \cdot 2 \cdot \sqrt{2 \cdot 3}$

$4\sqrt{6}$

f.) Simplify: $3\sqrt{8} \cdot 2\sqrt{5}$ → $6\sqrt{40}$

$\sqrt{40}$ → 2, 20 → 2, 10 → $\boxed{2}\boxed{5}$

2

$6 \cdot 2 \cdot \sqrt{2 \cdot 5}$

$12\sqrt{10}$

g.) Simplify: $\dfrac{2\sqrt{3}}{\sqrt{16}}$

$\dfrac{2\sqrt{3}}{4}$

$\dfrac{\sqrt{3}}{2}$

h.) Simplify: $\dfrac{\sqrt{2}}{3\sqrt{72}}$ → $\dfrac{1}{3} \cdot \dfrac{\sqrt{2}}{\sqrt{72}}$

$\dfrac{\sqrt{2}}{\sqrt{72}}$

↓

$\sqrt{\dfrac{2}{72}}$

$\sqrt{\dfrac{1}{36}} \rightarrow \dfrac{1}{6}$

$\dfrac{\sqrt{1}}{\sqrt{36}} = \dfrac{1}{6}$

$\dfrac{1}{3} \cdot \dfrac{1}{6}$

$1/18$

i.) Simplify: $\dfrac{12\sqrt{50}}{4\sqrt{2}}$ → $\dfrac{3\sqrt{50}}{\sqrt{2}}$

$\dfrac{\sqrt{50}}{\sqrt{2}}$

$\sqrt{\dfrac{50}{2}}$

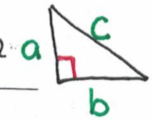
$\sqrt{25} \rightarrow 5$

$3 \cdot 5$

15

Review of the Pythagorean Theorem

Pythagorean Theorem → $a^2 + b^2 = c^2$ => $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$



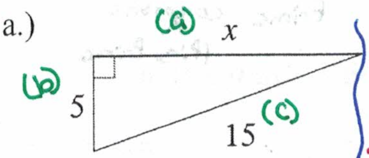
Remember → leg represents a side of a right triangle that forms the right angle

hypotenuse represents the side across from the right angle and is the longest side

*When finding missing sides → answers must be in SIMPLIFIED RADICAL FORM

Example 2: Find the length of the missing side x of each given right triangle. Keep in radical form.

a.)



$(x)^2 + (5)^2 = (15)^2$

$x^2 + 25 = 225$

$x^2 = 200$

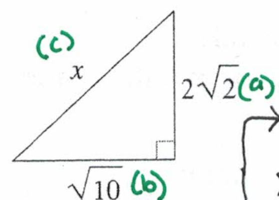
$x = \sqrt{200}$

$\sqrt{200}$: 2, 100; 2, 50; 2, 25; 5, 5

$2 \cdot 5 \cdot \sqrt{2}$

$10\sqrt{2}$

b.)



$(2\sqrt{2})^2 + (\sqrt{10})^2 = (x)^2$

$(2^2 \cdot (\sqrt{2})^2) + 10 = x^2$

$(4 \cdot 2) + 10 = x^2$

$8 + 10 = x^2$

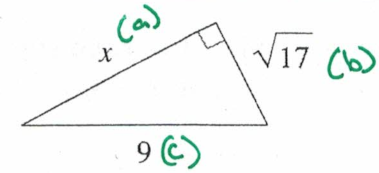
$18 = x^2$

$x = \sqrt{18}$

$\sqrt{18}$: 2, 9; 3, 3

$3\sqrt{2}$

c.)



$(x)^2 + (\sqrt{17})^2 = (9)^2$

$x^2 + 17 = 81$

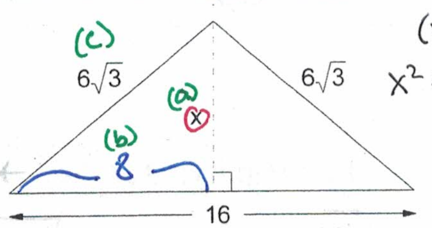
$x^2 = 64$

$x = 8$

8

Example 3 – Critical Thinking: Find the length of x. Round to tenth place.

a.)



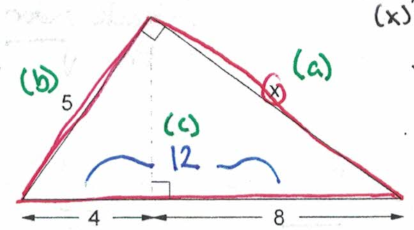
$(x)^2 + (8)^2 = (6\sqrt{3})^2$

$x^2 + 64 = 108$

$x^2 = 44$

$x \approx 6.6$

b.)



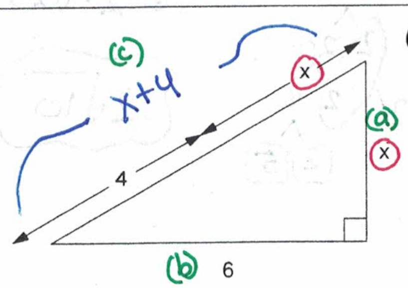
$(x)^2 + (5)^2 = (12)^2$

$x^2 + 25 = 144$

$x^2 = 119$

$x \approx 10.9$

c.)



$(x)^2 + (6)^2 = (x+4)^2$

$x^2 + 36 = x^2 + 8x + 16$

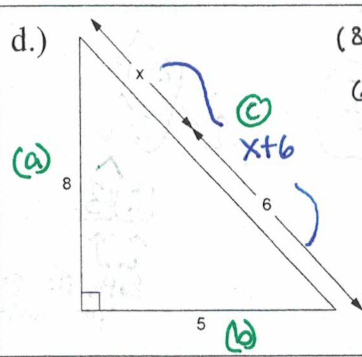
$36 = 8x + 16$

$20 = 8x$

$x = 20/8$

$x = 2.5$

d.)



$(8)^2 + (5)^2 = (x+6)^2$

$64 + 25 = x^2 + 12x + 36$

$89 = x^2 + 12x + 36$

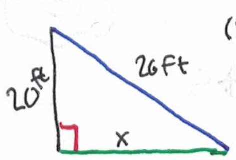
$0 = x^2 + 12x - 53$ ← Solve the quadratic

$a = 1 \quad b = 12 \quad c = -53$

$x = 3.4$

Example 4: For the following – a.) Draw a picture representing each word problem.
b.) Solve for what the problem is asking for. Round to tenth place.

a.) A telephone support cable attaches to the pole 20 feet high. If the cable is 26 feet long, how far from the bottom of the pole does the cable attach to the ground?



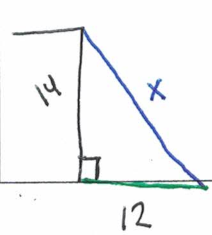
$(20)^2 + (x)^2 = (26)^2$

$400 + x^2 = 676$

$x^2 = 276$

$x \approx 16.6 \text{ ft}$

b.) Tara leaned a ladder against her house. The bottom of the ladder is 12 feet from the house and the top of the ladder is 14 feet above the ground. How long is the ladder?



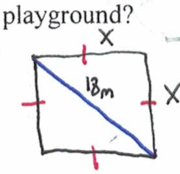
$(12)^2 + (14)^2 = x^2$

$144 + 196 = x^2$

$340 = x^2$

$x \approx 18.4 \text{ ft}$

c.) A walkway forms one diagonal of a square playground. The walkway is 18 meters long. How long are the sides of the playground?



$(x)^2 + (x)^2 = (18)^2$

$x^2 + x^2 = 324$

$2x^2 = 324$

$x^2 = 162$

$x \approx 12.7 \text{m}$

Properties of Radicals and Operations w/ Radicals

Product Property of Radicals

$$\sqrt{ab} \longleftrightarrow \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{24} \longrightarrow \sqrt{4} \cdot \sqrt{6}$$
$$2\sqrt{6}$$

$$3\sqrt{5} \cdot 2\sqrt{4} \rightarrow 6\sqrt{20}$$

20: 2, 10; 10: 2, 5

$$2 \cdot 6\sqrt{5}$$

$$12\sqrt{5}$$

Quotient Property of Radicals

$$\sqrt{\frac{a}{b}} \longleftrightarrow \frac{\sqrt{a}}{\sqrt{b}} \qquad b>0$$

* not allowed to have a radical in the denominator. *

$$\sqrt{\frac{13}{9}} \longrightarrow \frac{\sqrt{13}}{\sqrt{9}} \rightarrow \frac{\sqrt{13}}{3}$$

$$\frac{\sqrt{100}}{\sqrt{5}} \longrightarrow \sqrt{\frac{100}{5}} \rightarrow \sqrt{20} \rightarrow 2\sqrt{5}$$

Rationalizing the Denominator

(used to get rid of radicals in denominator)

$$\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

Need to make denominator into a perfect square!

$$\frac{\sqrt{13}}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{104}}{8} = \frac{2\sqrt{26}}{8} = \frac{\sqrt{26}}{4}$$

104: 2, 52; 52: 2, 26; 26: 2, 13; 2

$$\frac{\sqrt{13}}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{26}}{\sqrt{16}} \rightarrow \frac{\sqrt{26}}{4}$$

Conjugates to Rationalize

$a+b$ conjugate is $a-b$

$\sqrt{2}+5$ $\sqrt{2}-5$

$(\sqrt{2}+5)(\sqrt{2}-5)$

$2-5\sqrt{2}+5\sqrt{2}-25$

-23

$$\frac{2}{\sqrt{7}+3}\cdot\frac{\sqrt{7}-3}{\sqrt{7}-3} \quad \frac{2(\sqrt{7}-3)}{(\sqrt{7}+3)(\sqrt{7}-3)} \quad \frac{2\sqrt{7}-6}{-2} \rightarrow \frac{\sqrt{7}-3}{-1} \rightarrow -\sqrt{7}+3$$

$7-3\sqrt{7}+3\sqrt{7}-9$

$7-9$

-2

Example 3d)

$$x=\frac{-(12)\pm\sqrt{(12)^2-4(1)(-53)}}{2(1)} \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-12\pm\sqrt{356}}{2}$$

$$x=\frac{-12+\sqrt{356}}{2} \qquad x=\frac{-12-\sqrt{356}}{2}$$

$x=3.4$ $\qquad$ $x=-\cancel{15}.4$