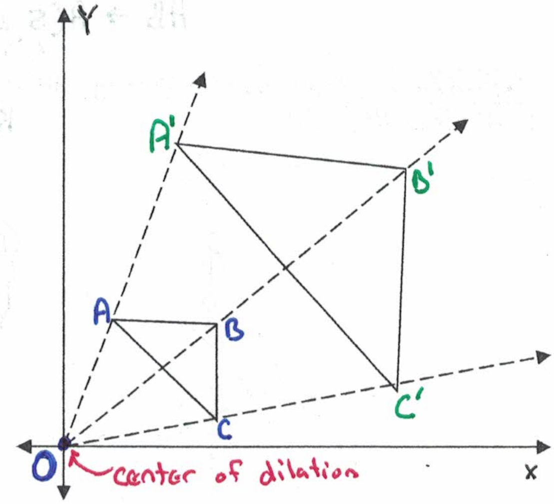
Math 2

Unit 4.4 Dilations

Dilation → or "scaling" is a *TRANSFORMATION* that ENLARGES or reduces a figure proportionally.



A dilation produces a Similar figure and is a type of similarity transformation.

IS NOT A RIGID TRANSFORMATION!

Dilations are performed with respect to a fixed point called the center of dilation.

The scale factor of a dilation decribes the extent of the dilation. The scale factor is the Ratio of a length on the image to a corresponding length on the PREIMAGE. The letter K is usually used to represent the scale factor of a dilation. The value of *"k"* determines whether the dilation is an ENLARGEMENT or a Reduction.

"K" = $\frac{\text{Image}}{\text{Preimage}}$

Dilation Symbolism: $D_{O,2}$ dilation, scale factor "K"

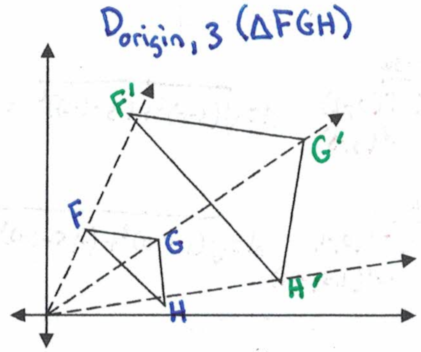
Fixed point for the center of dilation

means a dilation Centered on O with a scale factor of 2.

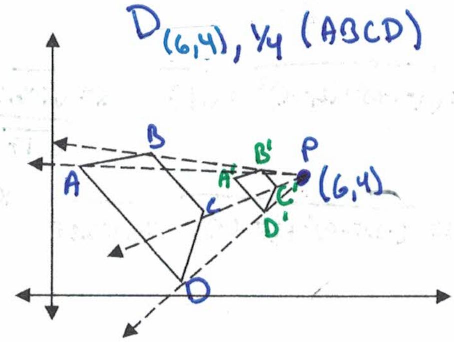
- There are 2 types of dilations:

<table>
<tr><td>1.) ENLARGEMENTS</td><td>2.) REDUCTION</td></tr>
<tr><td>A dilation that has a scale factor greater than 1 that produces an image that is larger than the preimage.

If k > 1, then the dilation is an enlargement.

$D_{origin,3}$ (ΔFGH)
</td><td>A dilation that has a scale factor between 0 and 1 that produces an image that is smaller than the preimage.

If 0 < k < 1, then the dilation is a reduction.

$D_{(6,4), 1/4}$ (ABCD)
</td></tr>
</table>

ΔFGH is dilated by a scale factor of 3 to produce ΔF'G'H'. Since 3>1, ΔF'G'H' is an enlargement.

ABCD is dilated by a scale factor of 1/4 to produce A'B'C'D'. Since 0<1/4<1, A'B'C'D' is a reduction.

FYI: Distance Formula (used on the coordinate plane to find the distance between two points)

* will use this to find the scale factor "K"!

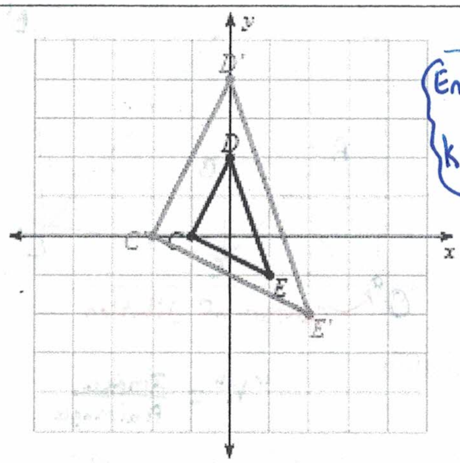
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\overline{AB} \rightarrow A(\underset{x_1}{5},\underset{y_1}{2})\quad B(\underset{x_2}{-3},\underset{y_2}{7}) \rightarrow d=\sqrt{(-3-5)^2+(7-2)^2} = d=\sqrt{89} \approx 9.4$

Example 1: Determine whether the dilation from "A" to "B" is an *enlargement* or a *reduction*. Then find the scale factor.

K = the ratio of $\frac{\text{IMAGE}}{\text{PREIMAGE}}$

a)



Enlargement,

k=2

Preimage

$\overline{CD}$: C(-1,0) D(0,2) $d=\sqrt{(0-(-1))^2+(2-0)^2} = \sqrt{5}$

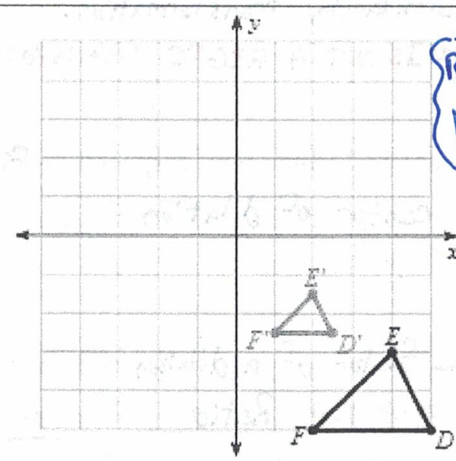
Image

$\overline{C'D'}$ C'(-2,0) D'(0,4) $d=\sqrt{(0-(-2))^2+(4-0)^2} = \sqrt{20}$

$K=\frac{\sqrt{20}}{\sqrt{5}}, \sqrt{\frac{20}{5}}$

$k=\sqrt{4}$

b)



Reduction,

$K = 1/2$

Preimage

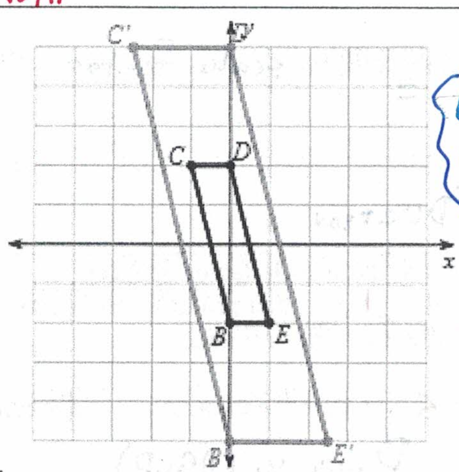
$\overline{FD}$ F(2,-5) D(5,-5) $d=\sqrt{(5-2)^2+(-5-(-5))^2} = \sqrt{9} = 3$

Image

$\overline{F'D'}$ F'(1,-2.5) D'(2.5,-2.5) $d=\sqrt{(2.5-1)^2+(-2.5-(-2.5))^2} = \sqrt{2.25} = 1.5$

$K = \frac{1.5}{3} = 1/2$

c)



ENLARGEMENT

K=2.5

Preimage

$\overline{BC}$: B(0,-2) C(-1,2) $d=\sqrt{(-1-0)^2+(2-(-2))^2} = \sqrt{17}$

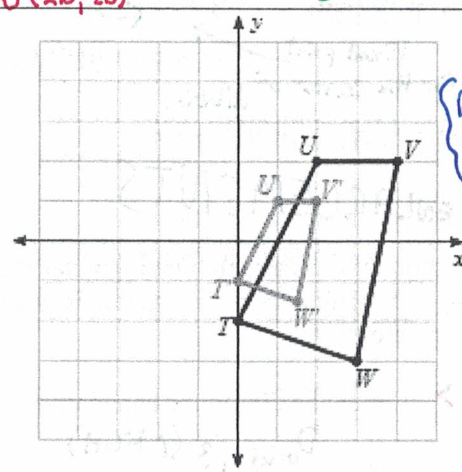
$K=\frac{\sqrt{106.25}}{\sqrt{17}}$

Image

$\overline{B'C'}$: B'(0,-5) C'(-2.5,5) $d=\sqrt{(-2.5-0)^2+(5-(-5))^2} = \sqrt{106.25}$

K=2.5

d)



Reduction

K=1/2

Preimage

$\overline{TW}$: T(0,-2) W(3,-3) $d=\sqrt{(3-0)^2+(-3-(-2))^2} = \sqrt{10}$

$K=\frac{\sqrt{2.5}}{\sqrt{10}}$

Image

$\overline{T'W'}$: T'(0,-1) W'(1.5,-1.5) $d=\sqrt{(1.5-0)^2+(-1.5-(-1))^2} = \sqrt{2.5}$

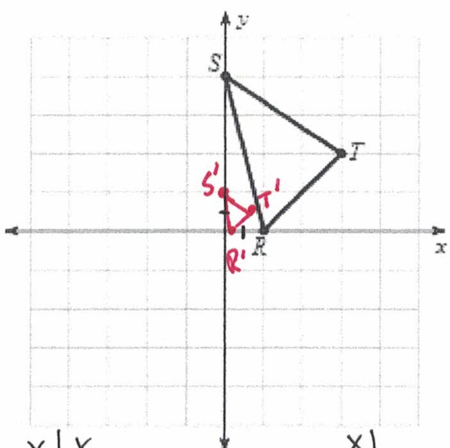
K=1/2

The scale factor of a dilation is used as a <u>Scaler</u> to perform dilations centered on the origin on a figure in the coordinate plane. *(The figure is not necessarily centered on the origin.)*

To find the coordinates of an image after a dilation centered at the origin, <u>MULTIPLY</u> the **x**- and **y**-coordinates of each point of <u>the preimage</u> by the scaler (scale factor) of the dilation.

<u>Example 2</u>: Given the preimage or the coordinates of its vertices, find the vertices of the image after the stated dilation centered at the origin and graph the image.

a) $D_{\text{Origin}, .25}$ * $K = 1/4$ means reduction

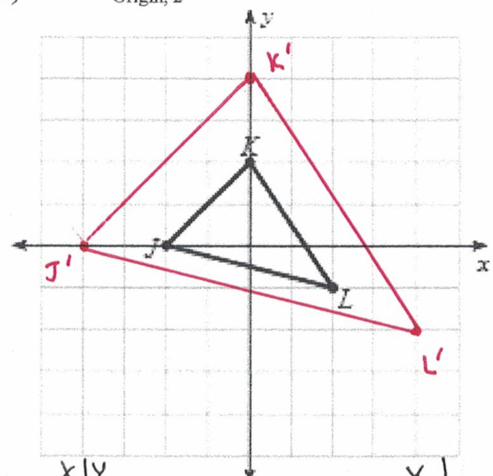


	X	Y
R	1	0
S	0	4
T	3	2

$D_{\text{origin}, 1/4}$

X	Y	
$1/4(1) = 1/4$	$1/4(0) = 0$	$R'(1/4, 0)$
$1/4(0) = 0$	$1/4(4) = 1$	$S'(0, 1)$
$1/4(3) = 3/4$	$1/4(2) = 1/2$	$T'(3/4, 1/2)$

b) $D_{\text{Origin}, 2}$



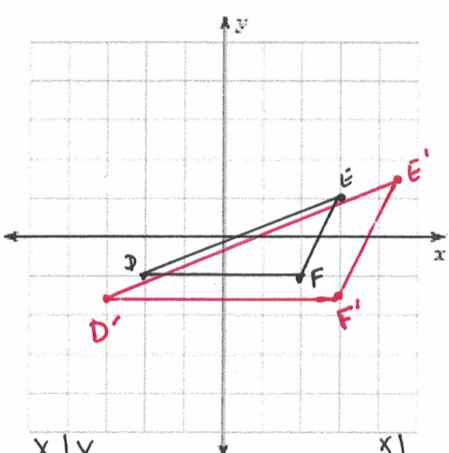
	X	Y
J	-2	0
K	0	2
L	2	-1

$D_{\text{origin}, 2}$

X	Y	
$2(-2) = -4$	$2(0) = 0$	$J'(-4, 0)$
$2(0) = 0$	$2(2) = 4$	$K'(0, 4)$
$2(2) = 4$	$2(-1) = -2$	$L'(4, -2)$

c) $D_{\text{Origin}, 1.5}$ $K = 1.5$ means enlargement

$D(-2, -1), E(3, 1), F(2, -1)$



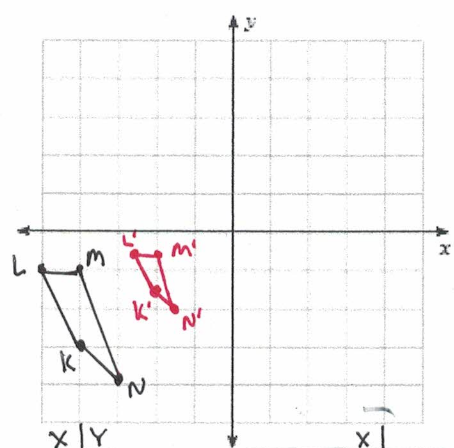
	X	Y
D	-2	-1
E	3	1
F	2	-1

$D_{(0,0), 1.5}$

X	Y	
$1.5(-2) = -3$	$1.5(-1) = -1.5$	$D'(-3, -1.5)$
$1.5(3) = 4.5$	$1.5(1) = 1.5$	$E'(4.5, 1.5)$
$1.5(2) = 3$	$1.5(-1) = -1.5$	$F'(3, -1.5)$

d) $D_{\text{Origin}, 0.5}$

$K(-4, -3), L(-5, -1), M(-4, -1), N(-3, -4)$



	X	Y
K	-4	-3
L	-5	-1
M	-4	-1
N	-3	-4

$D_{\text{origin}, 1/2}$

X	Y	
$1/2(-4) = -2$	$1/2(-3) = -1.5$	$K'(-2, -1.5)$
$1/2(-5) = -2.5$	$1/2(-1) = -1/2$	$L'(-2.5, -.5)$
$1/2(-4) = -2$	$1/2(-1) = -1/2$	$M'(-2, -.5)$
$1/2(-3) = -1.5$	$1/2(-4) = -2$	$N'(-1.5, -2)$