4.6 – Piecewise Functions

– **piecewise function** → a function and its graph is broken into different parts (hence its name) and different functions are ***defined on VARIOUS DOMAIN VALUES*** (x-values).

x-values tell you what part to use!

$\boxed{\text{Step \# 1}}$ → Evaluate the function based on its domain (x) values ; start with #'s after the word IF.

$\boxed{\text{Step \# 2}}$ → Make a table of values (points) ; indicate if those points are closed or open dots.
$= \ \le \ \ge \qquad \ne \ < \ >$

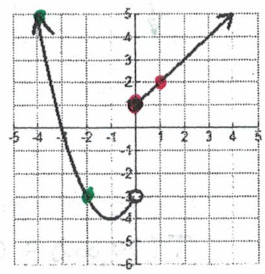
$\boxed{\text{Step \# 3}}$ → After graphing (by hand – calculator is not helpful), CHECK for one MAJOR FEATURE: Piecewise function's graphs should NOT CROSS EACH OTHER!

Example 1: Evaluate each piecewise function. Validate points with the function's given graph.

Given Piecewise Function	Graph of Piecewise Function	Evaluate/Complete Table of Values

a.)

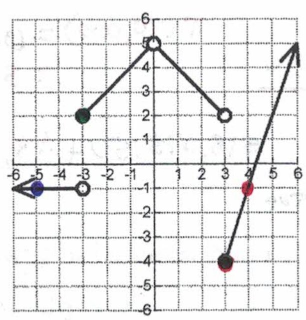
$$f(x)=\begin{cases} x^2+2x-3 & \text{if} \quad x<0 \\ x+1 & \text{if} \quad x\ge 0 \end{cases}$$



x	Work to find f (x) or y	Pt (x,y)
– 4	$(-4)^2+2(-4)-3=5$	(-4,5)
– 2	$(-2)^2+2(-2)-3=-3$	(-2,-3)
0	(0)+1 = 1	(0,1)
1	(1)+1 = 2	(1,2)

b.) f(x)=-1 → y=-1 (horizontal line)

$$f(x)=\begin{cases} -1 & \text{if} \quad x<-3 \\ -|x|+5 & \text{if} \quad -3\le x<3 \cup x\ne 0 \\ 3x-13 & \text{if} \quad x\ge 3 \end{cases}$$



x	Work to find f (x) or y	Pt (x,y)
4	3(4)-13 = -1	(4,-1)
– 3	-|-3| +5 = 2	(-3,2)
– 5	-1	(-5,-1)
3	3(3)-13 = -4	(3,-4)
0	-|0|+5=5 Undefined	∅

Example 2: Make a table of domain values ("work") and graph each piecewise function.

a.)

$$f(x)=\begin{cases} 2x+3 & \text{if} \quad x<-1 \\ 3-x & \text{if} \quad x\ge -1 \end{cases}$$

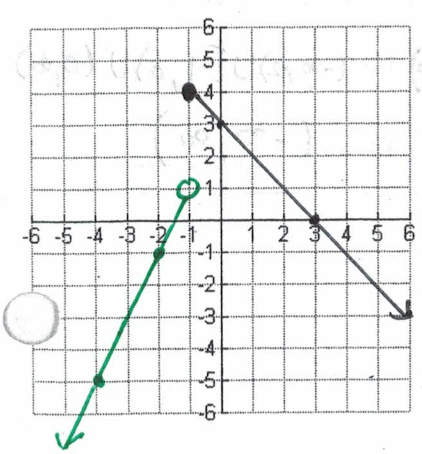


Table of Values

2x+3	3-x
x=-1 (-1,1) 2(-1)+3=1	x=-1 (-1,4) 3-(-1)=4
x=-2 (-2,-1) 2(-2)+3=-1	x=0 (0,3) 3-(0)=3
x=-4 (-4,-5) 2(-4)+3=-5	x=3 (3,0) 3-(3)=0
	x=6 (6,-3) 3-(6)=-3

b.)

$$f(x)=\begin{cases} \frac{1}{2}|x|-4 & \text{if} \quad x\ne 0 \\ 3 & \text{if} \quad x=0 \end{cases}$$

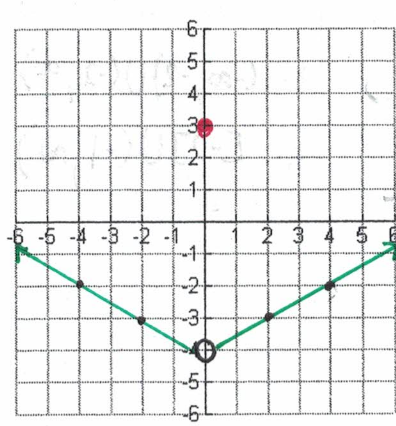


Table of Values

½|x|-4	3
x=0 (0,-4) ∅	x=0 (0,3)
x=2 (2,-3) ½|2|-4=-3	
x=-2 (-2,-3) ½|-2|-4=-3	
x=4 (4,-2) ½|4|-4=-2	

x=-4 (-4,-2)
½|-4|-4=-2

Example 2 Cont'd: Make a table of domain values ("work") and graph each piecewise function.

c.)
$$f(x)=\begin{cases}3 & \text{if } x\le -2\\ 2x-1 & \text{if } -2<x\le 1\\ 4-x & \text{if } x>1\end{cases}$$

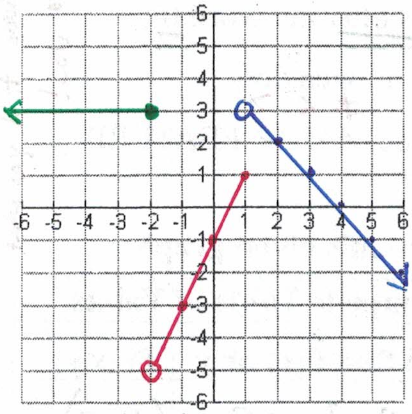


Table of Values		
3	2x-1	4-x
x=-2 (-2,3)	x=-2 (-2,-5)	x=1 (1,3)
x=-3 (-3,3)	x=-1 (-1,-3)	x=2 (2,2)
x=-4 (-4,3)	x=0 (0,-1)	x=3 (3,1)
x=-5 (-5,3)	x=1 (1,1)	x=4 (4,0)

d.)
$$f(x)=\begin{cases}-(x+2)^2+4 & \text{if } x<-1 \cup x\neq -2\\ 1 & \text{if } x=-2\\ 2\sqrt{x+1}-2 & \text{if } x>-1\end{cases}$$

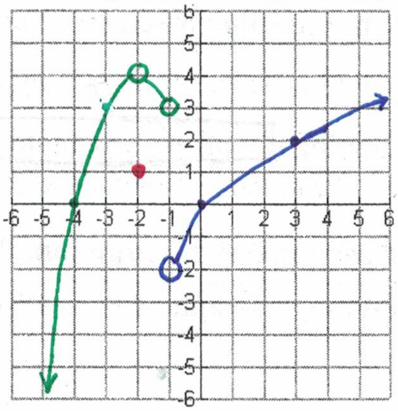


Table of Values		
$-(x+2)^2$	1	$2\sqrt{x+1}-2$
x=-1 (-1,3)	x=-2 (-2,1)	x=-1 (-1,-2)
x=-2 ∅ (-2,4)		x=0 (0,0)
x=-3 (-3,3)		x=3 (3,2)
x=-4 (-4,0)		x=8 (8,4)

Example 3: Complete the problem.

During a particular year, the taxes owed by a married person filing separately with an adjusted gross income of x dollars is given by the piecewise function below:

$$T(x)=\begin{cases}0.15x & \text{if } 0\le x<17{,}900\\ 0.28(x-17{,}900)+2685 & \text{if } 17{,}900\le x<43{,}250\\ 0.31(x-43{,}250)+9783 & \text{if } x\ge 43{,}250\end{cases}$$

Find and interpret: T (70,000) + T (40,000)

T(70,000) = 0.31(70,000 − 43,250) + 9783
= $18,075.50

T(40,000) = 0.28(40,000 − 17,900) + 2685
= $8,873

$18,075.50 + $8,873 = $26,948.50 the adjusted gross income that the couple had to pay taxes on.

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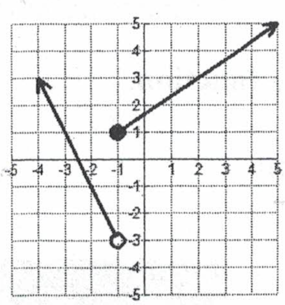
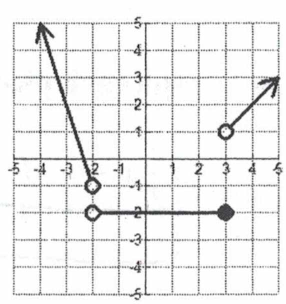
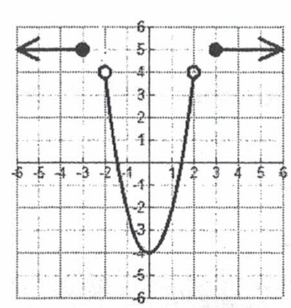
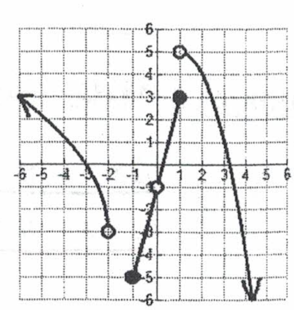
Example 4: Determine the domain and range of each piecewise graph in interval notation.

Example 4a	Example 4b	Example 4c	Example 4d
			
D: (-∞, ∞)	D: (-∞,-2)∪(-2,∞)	D: (-∞,-3]∪(-2,2)∪[3,∞)	D: (-∞,-2)∪[-1,0)∪(0,∞)
R: (-3, ∞)	R: [-2]∪(-1,∞)	R: [-4,4)∪[5]	R: (-∞,∞)