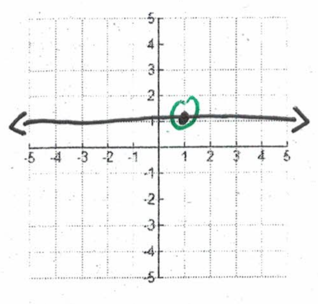
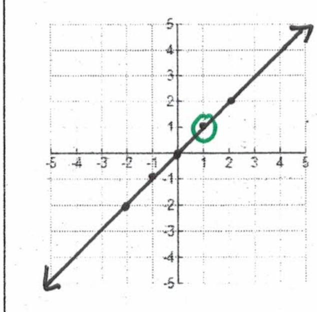
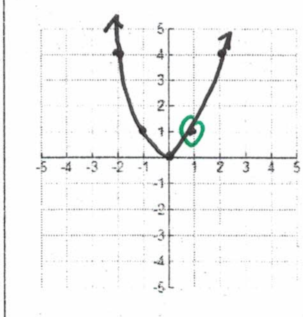
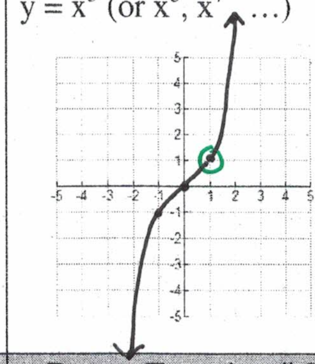
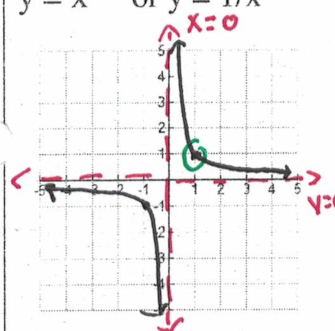
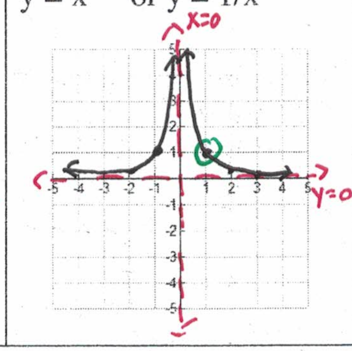
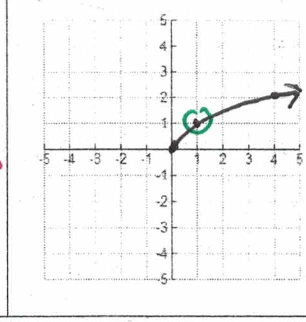
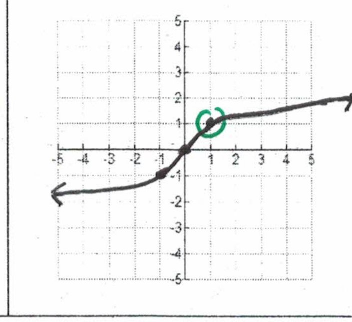
4.5 – Power Functions and Equations

Power Functions and Their Characteristics

– **power function** → a function in the form $y = Kx^p$ (k and p are constants) where k is called the constant of proportionality and p is the exponent (p can be any real value, rational/irrational, or positive/negative)

Most Parent Functions are Power Functions

Below are some examples of power functions where k = 1 and various values for p:

Power Function # 1	Power Function # 2	Power Function # 3	Power Function # 4
$y = x^0 \rightarrow y = 1$ 	$y = x^1$ 	$y = x^2$ (or x^4, x^6, …) 	$y = x^3$ (or x^5, x^7, …) 
Power Function # 5	**Power Function # 6**	**Power Function # 7**	**Power Function # 8**
$y = x^{-1}$ or $y = 1/x$ 	$y = x^{-2}$ or $y = 1/x^2$ 	$y = x^{½}$ or $y = \sqrt{x}$ 	$y = x^{⅓}$ or $y = \sqrt[3]{x}$ 

Fractional Exponent is a radical!

Example 1: Determine which are power functions, circle YES or NO. If YES, state value of k and p.

SEE ATTACHED WORK

a.) $f(x) = 5\sqrt[3]{x^{12}}$ power function? Circle one: YES NO where k = 5 and p = 4

b.) $f(x) = 6(x-1)^2$ power function? Circle one: YES NO where k = N/A and p = N/A

c.) $f(x) = \sqrt{\dfrac{36}{x^5}}$ power function? Circle one: YES NO where k = 6 and p = -5/2

d.) $10y + 2 = 5x^4 + 2$ power function? Circle one: YES NO where k = 1/2 and p = 4

e.) $f(x) = -5 \cdot 2^x$ power function? Circle one: YES NO where k = N/A and p = N/A

f.) $¼\, y = (x-3)(x+3) + 9$ power function? Circle one: YES NO where k = 4 and p = 2

g.) $y + 3 = 3(x + 1)$ power function? Circle one: YES NO where k = 3 and p = 1

Example 2: Find the equation of a power function with the given information.

Power Function in the form $y = k \cdot x^p$ where the point (x , y) and the point (1 , ?) (k = ?) are on the graph		
a.) pts (2 , 12) ; (1 , 4) $k=4$ $y=kx^p$ $y=4x^p$ $12=4(2)^p$ $3=2^p$ $\log 3=\log 2^p$ $\log 3=(p)\log 2$ $p\approx 1.585$ $\boxed{y=4x^{1.585}}$	b.) pts (7 , 9) ; (1 , ½) $k=\frac{1}{2}$ $y=kx^p$ $y=\frac{1}{2}x^p$ $9=\frac{1}{2}(7)^p$ $18=7^p$ $\log 18=p\log 7$ $p=\frac{\log 18}{\log 7}$ $p\approx 1.4854$ $\boxed{y=\frac{1}{2}x^{1.4854}}$	c.) pts (4 , 0.375) and (9 , 0.25) * SEE ATTACHED NOTES *

Solving Power Equations – Direct and Inverse Variations

Direct Variation Equation: $y=kx^p$ (where p is a positive #)	**Inverse Variation Equation:** $y=\frac{k}{x^p}$ or $y=kx^{-p}$ (where p is a negative #)

Example 3: Complete each variation problem.

a.) Suppose y is directly proportional to x. If y = 18 when x = 8, find the constant of proportionality (k). After finding the formula for y, then use it to find x when y = 27. $y=kx^p$ ① $18=k(8)^1$ $\boxed{k=2.25}$ ② $y=2.25x$ ③ $27=2.25x^1$ $\boxed{x=12}$	b.) Suppose c is inversely proportional to the square of d. If c = 4 when d = 2, find the constant of proportionality (k). After finding the formula for c, then use it to find c when d = – 8. $y=\frac{k}{x^p}$ $c=\frac{k}{d^2}$ ① $4=\frac{k}{(2)^2}$ $4=\frac{k}{4}$ $\boxed{k=16}$ ② $c=\frac{16}{d^2}$ ③ $c=\frac{16}{(-8)^2}$ $c=\frac{16}{64}$ $\boxed{c=\frac{1}{4}}$

c.) The radius of a sphere is directly proportional to the cube root of its volume. If a sphere of radius 18.2 cm has a volume of 25,252.4 cm^3, what is the volume of a sphere if the radius is 19.3 cm? $y=kx^p$

$r=k\sqrt[3]{V}$

$18.2=k\sqrt[3]{25{,}252.4}$

$18.2=k(29.3383)$

$\frac{18.2}{29.3383} = \frac{k(29.3383)}{29.3383}$

$\boxed{k\approx .6203}$

$r=.6203\sqrt[3]{V}$

$\frac{19.3}{.6203}=\frac{.6203\sqrt[3]{V}}{.6203}$

$31.114=\sqrt[3]{V}$

$(31.114)^3=(\sqrt[3]{V})^3$

$\boxed{V=30{,}120.8721 \text{ cm}^3}$

Ex. 1

a) $f(x) = 5\sqrt[3]{x^{12}}$

$y = kx^p$

rewrite the radical with fractional exponents!

$f(x) = 5x^{\frac{12}{3}}$

$f(x) = 5x^4$

Yes; $k = 5$, $p = 4$

b) $f(x) = 6(x-1)^2$ ← You must box/FOIL then distribute the 6!

$f(x) = 6(x^2 - 2x + 1)$

$f(x) = 6x^2 - 12x + 6$ → This is why it is not a power function!

$y = kx^p$

No; $k =$ N/A $p =$ N/A

c) $f(x) = \sqrt{\frac{36}{x^5}}$

$y = kx^p$

$f(x) = \sqrt{36x^{-5}}$

$f(x) = (36x^{-5})^{1/2}$

$f(x) = 6x^{-5/2}$

Rewrite with fractional exponents $\sqrt{\ } \rightarrow (\ )^{1/2}$

$36^{1/2}(x^{-5})^{1/2}$

$6x^{-5/2}$

Yes; $k = 6$ $p = -5/2$

d) $10y + 2 = 5x^4 + 2$

$y = kx^p$

$10y = 5x^4$

$y = \frac{5x^4}{10}$

$y = \frac{1}{2}x^4$

Yes; $k = 1/2$ $p = 4$

e) $f(x) = -5 \cdot 2^{x}$ (Exponential)

$y = K \cdot x^{p}$

No; $K = N/A$ $p = N/A$

$\boxed{\text{X is not the base!}}$

f) $\frac{1}{4}y = (x-3)(x+3) + 9$ $y = kx^{p}$

$\frac{1}{4}y = x^2 - 9 + 9$

$(4)\frac{1}{4}y = x^2(4)$

$y = 4x^2$ Yes; $K = 4$ $p = 2$

g) $y + 3 = 3(x+1)$

$y + 3 = 3x + 3$

$y = 3x$ Yes; $K = 3$ $p = 1$

Ex. 2

c) pts $(4, .375)$ and $(9, .25)$ $\boxed{y = \frac{3}{4}x^{-\frac{1}{2}}}$

$y = kx^{p}$

① Find k * Take ONE ORDERED PAIR AND SOLVE FOR k! * ② Find P

$.375 = k(4)^{p}$

Substitute this "k" expression into the other ordered pair!

$.25 = K(9)^{p}$

$K = \frac{.375}{4^{p}}$

$.25 = \left(\frac{.375}{4^{p}}\right)9^{p} \rightarrow .375\left(\frac{9^{p}}{4^{p}}\right) \rightarrow .375\left(\frac{9}{4}\right)^{p}$

Substitute "p" here!

$\frac{.25}{.375} = \frac{.375}{.375}\left(\frac{9}{4}\right)^{p}$

$K = \frac{.375}{4^{-1/2}}$

$\frac{2}{3} = \left(\frac{9}{4}\right)^{p}$

$K = \frac{3}{4}$ $p = -\frac{1}{2}$

$\log\frac{2}{3} = \log\left(\frac{9}{4}\right)^{p} \rightarrow \log\frac{2}{3} = p\log\left(\frac{9}{4}\right)$