4.4 – Exponential/Logarithmic Word Problems

Exp Growth	Exp Decay	Comp'd With n values	Comp'd Contin'ly
$A = a(1+r)^t$ A = final amount a = initial amount r = rate of growth % t = time Key words: Increase Grows Appreciates Change to a decimal!	$A = a(1-r)^t$ A = final amount a = initial amount r = rate of decay % t = time Key words: Decrease Decays Depreciates Change to a decimal!	Compound interest $A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$ A = final amount P = principle amount r = interest rate % n = # of times \$ is comp'd t = time (always in years) annually: n = 1 · monthly: n = 12 semiannually: n = 2 · weekly: n = 52 quarterly: n = 4 · daily: n = 365	$A = Pe^{r\cdot t}$ A = final amount P = principle amount r = interest rate e = exp function natural base is not a variable Key word: Continously

Example 1: Complete each exponential word problem.

a.) You bought a car for \$24,000. The car's value has depreciated by 8.7% each year. How much will your car be worth 11 years from initially buying it?

$A = a(1-r)^t$

A: r: 8.7 → .087

a: 24,000   t: 11

A = \$8,818.40 will be the car's value after 11 yrs!

b.) In 1910, the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in 2014?

$A = a(1+r)^t$

A: ?   r: 1.5% → .015

a: 120,000   t: 2014 − 1910 = 104

A = 564,481 people is the population

c.) An island initially had 500 rabbits and is growing each year. After 16 years, there are 45,000 rabbits that inhabit the island. What is the growth rate of the rabbits on the island?

r = ?

$A = a(1+r)^t$

A: 45,000   r: ?

a = 500   t: 16

Growth rate is 32.5%.

d.) Amber has a savings account in which her money is being compounded continuously with a 3% interest rate. After 8 years, Amber's account has a balance of \$1,907. What was Amber's initial deposit for the account?

$A = Pe^{rt}$

A: \$1907   r: 3% → .03

P: t: 8

Initial deposit was \$1500.10

e.) Mike decides to invest \$400 into an account that has a 6% interest rate.

i.) What is the balance in the account after 4 years if the account is being compounded monthly?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

A: r: 6% → .06   t: 4

P: 400   n: 12

\$508.20

ii.) What is the balance in the account after 4 years if the account is being compounded continuously?

$A = Pe^{rt}$

P = 400

r = .06

t = 4

\$508.50

f.) Desmond is investing \$800 into an account with a 5% interest rate.

i.) How long will it take for the account to be \$2800 if the money is compounded quarterly?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

A = 2800   r = 5% → .05   t = ?

P = 800   n = 4

25.2 yrs.

ii.) How long will it take for the account to double if the money is compounded continuously?

$A = Pe^{rt}$

A = 1600   r = .05

P = 800   t = ?

13.9 yrs

4.4

Example 1

1a) $A = 24000(1-.087)^{11}$

$A = \$8,818.40$

1b) $A = 120,000(1+.015)^{104}$

$A = 564,481$ people

* Don't forget units and round to the nearest cent!

1c) $\frac{45,000}{500} = \frac{500(1+r)^{16}}{500}$

$90 = (1+r)^{16}$

$\sqrt[16]{90} = \sqrt[16]{(1+r)^{16}}$

$1.324769 = 1+r$

$r = .3247689997$ Needs to be changed to a % to the nearest tenth.

$r = 32.5\%$

1d) This is not a variable!

$1907 = Pe^{.03(8)}$

$\frac{1907}{e^{.24}} = \frac{Pe^{.24}}{e^{.24}}$

$P = \$1500.10$

1e)

i) $A = 400\left(1+\frac{.06}{12}\right)^{12(4)}$

$A = 400(1.005)^{48}$

$A = \$508.20$

1e)

ii) $A = 400e^{.06(4)}$

$A = \$508.50$

1f)

i) $2800 = 800\left(1+\frac{.05}{4}\right)^{4t}$

$\frac{2800}{800} = \frac{800(1.0125)^{4t}}{800}$

$3.5 = (1.0125)^{4t}$

$\log 3.5 = \log 1.0125^{4t}$ Power Property

$\frac{\log 3.5}{\log 1.0125} = \frac{(4t)\log 1.0125}{\log 1.0125}$

$\frac{100.8461221}{4} = \frac{4t}{4}$ $t = 25.2$ yrs

1f)

ii) $\frac{1600}{800} = \frac{800e^{.05t}}{800}$

$2 = e^{.05t}$

$\ln 2 = \cancel{\ln e}^{.05t}$

$\frac{\ln 2}{.05} = \frac{.05t}{.05}$

$t = 13.9$ yrs

Log Scale – pH Scale	Log Scale – Decibel Scale	Log Scale – Memory Recall
$pH = -\log(H)$ pH = acidity of a solution If pH < 7 then solution is acidic If pH = 7 then solution is neutral If pH > 7 then solution is basic H = hydrogen ions in M where H has to be in scient. not.	$D = 10(\log I + 12)$ D = intensity level in dB (decibels) I = intensity of any given sound where measures in W/m^2 (Watts/meters^2)	$R = 75 - [6\ln(t+1)]$ R = percent of the info retained t = number of months that have gone by after being presented with info.

scientific notation

3.7×10^{-8} 2.5×10^{3}

Example 2: Complete each logarithmic word problem.

a.) The hydrogen ion of a sample of human blood was measured to be $H = 3.16\times10^{-8}$ M. Find the pH and classify the sample. $pH = -\log(H)$ pH: $H: 3.16\times10^{-8}$ $pH = -\log(3.16\times10^{-8})$ pH = 7.5 and it is basic.	c.) A jet engine during takeoff has an intensity measured at 100 W/m^2. What is the jet engine's intensity level? $D = 10(\log I + 12)$ $D = 10(\log 100 + 12)$ D = 140 dB	e.) What percent of memory was retained 6 months after being presented the information? $R = 75 - [6\ln(t+1)]$ $t = 6$ $R = 75 - [6\ln(6+1)]$ $R = 75 - [6\ln(7)]$ R = 63.3%
b.) The most acidic rainfall ever measured occurred in Scotland in 1974, its pH = 3.8. What is the hydrogen ion concentration of this rainfall? $pH = -\log(H)$ pH = 3.8 H: ? $\frac{3.8}{-1} = \frac{-\log(H)}{-1}$ $-3.8 = \log_{10} H$ Rewrite as exponential $H = 10^{-3.8}$ $H = 1.6\times10^{-4}$ M	d.) The intensity level of sound of a subway train was measured to be 98 dB. What is the intensity? $D = 10(\log I + 12)$ $\frac{98}{10} = \frac{10(\log I + 12)}{10}$ $9.8 = \log I + 12$ $-12 \quad -12$ $-2.2 = \log_{10} I$ rewrite as an exponential $10^{-2.2} = I$ I = .0063095734 6.3×10^{-3} w/m^2	f.) After how many months did the average person retain only half of the presented information? R = 50 t = ? $50 = 75 - [6\ln(t+1)]$ $-75 \quad -75$ $\frac{-25}{-1} = \frac{-[6\ln(t+1)]}{-1}$ $\frac{25}{6} = \frac{6\ln(t+1)}{6}$ $\frac{25}{6} = \cancel{\ln}(t+1)$ e e $e^{25/6} = t+1$ $-1 \quad -1$ $e^{25/6} - 1 = t$ t = 63.5 months