3.3 Rate of Change (ROC) and Slope

Rate of Change (ROC)

→ Will have 2 different Units of Measures.

*- **rate of change** → (allows you) to see the relationship between two different QUANTITIES that are Changing.

- if given a table → ROC = $\dfrac{\text{(-) Change of the Dependent Variable (Y)}}{\text{Change of the Independent Variable (X)}}$ - Dependent is RIGHT COLUMN; - Independent is LEFT COLUMN
- if given a graph → ROC = $\dfrac{\text{vertical change Y}}{\text{horizontal change X}}$

FYI - Δ "delta" it means "the change in".

Example 1: Find the rate of change.

a.) table

(X) Independ. Tickets	(Y) Dependent Cost
5	$75
6	$90
7	$105
8	$120

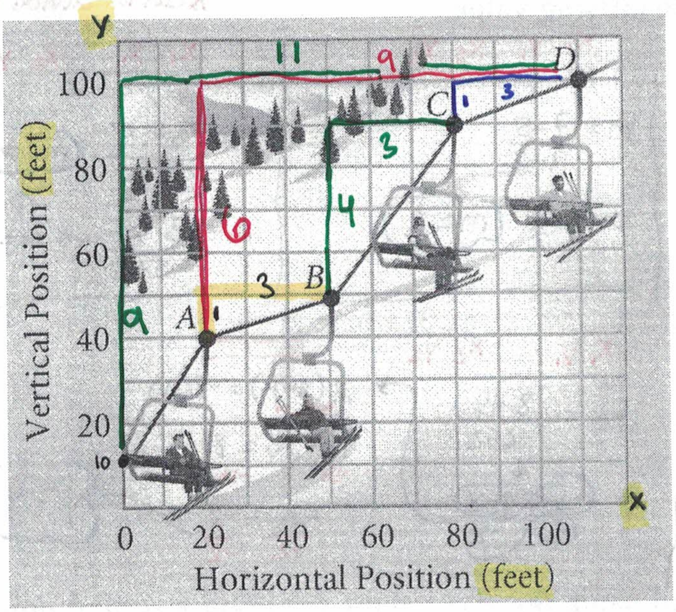
ROC b/w 5 and 6 tickets → $\dfrac{\Delta Y \text{ (Dependent)}}{\Delta X \text{ (Independent)}}$ → $\dfrac{\$90 - \$75}{6 - 5} = \dfrac{\$15}{1 \text{ ticket}}$ $15 per ticket

ROC b/w 6 and 7 tickets → $\dfrac{\$105 - \$90}{7 - 6}$ → $\dfrac{\$15}{1 \text{ ticket}}$ $15 per ticket

ROC b/w 7 and 8 tickets → $\dfrac{\$120 - \$105}{8 - 7}$ → $\dfrac{\$15}{1 \text{ ticket}}$ $15 per ticket

What does the ROC mean in this example? each ticket cost $15.

b.)



ROC b/w A to B → $\dfrac{\Delta \text{ vertical}}{\Delta \text{ Horizontal}}$ → $\dfrac{1}{3}$ for every 1 ft of vertical change you have 3 ft horizontal change.

ROC b/w B to C → $\dfrac{4}{3} \rightarrow 1.\overline{3}$

ROC b/w C to D → $\dfrac{1}{3} \rightarrow .\overline{3}$

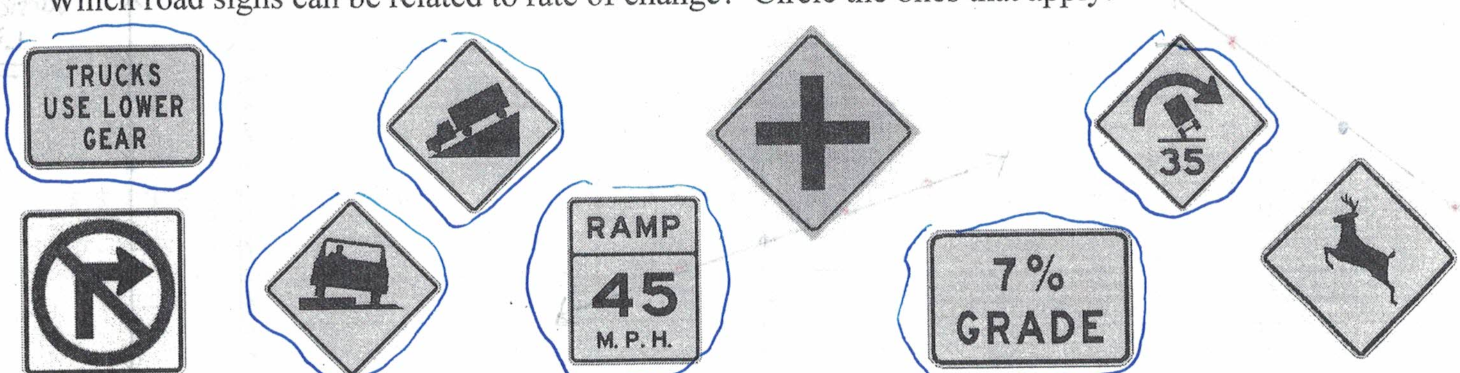
ROC b/w A to D → $\dfrac{6}{9} \xrightarrow{\text{REDUCE}} \dfrac{2}{3} \rightarrow .\overline{6}$

ROC b/w bottom of lift to D → $\dfrac{9}{11} \rightarrow .\overline{81}$

Which section is the steepest? between B to C

Explain: B to C has the highest ratio for the rate of change.

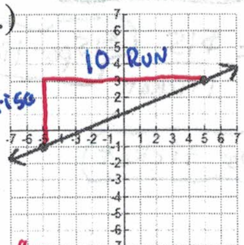
c.) Road signs are a real world example of where rate of change (and slope) are seen everyday. Which road signs can be related to rate of change? Circle the ones that apply.



(lower case)

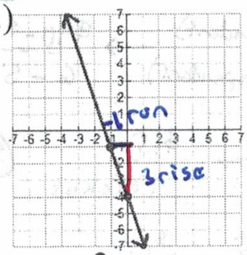
- **slope** → represented by the letter m and can be found in three ways:
  - graph by using $\frac{\Delta \text{vertical}}{\Delta \text{horizontal}}$ $\frac{\Delta y}{\Delta x}$ $\frac{\text{RISE}}{\text{RUN}}$
  - * two points by using the formula $m=\frac{y_2-y_1}{x_2-x_1}$ $A(3,8)$ $B(-5,7)$ ($x_1\ y_1$, $x_2\ y_2$)
  - ______ which we will do later (but not right now)… $m=\frac{7-8}{-5-3}$ LEAVE AS A FRACTION!

Example 2: Find the slope of each graph and describe the type of line given.

a.)


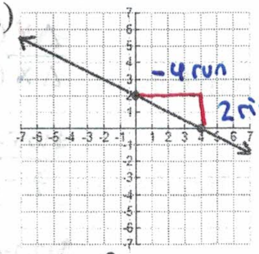
Reduce $m=\frac{4}{10} \rightarrow m=\frac{2}{5}$

Rises to the Right (Falls to the Left)

b.)


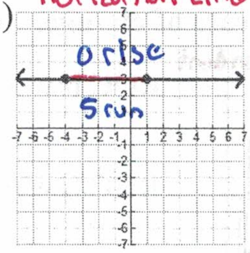
$m=\frac{3}{-1} \rightarrow m=-3$

Falls to the Right (Rises to the Left)

c.)


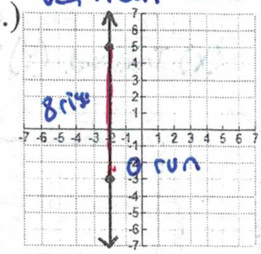
$m=\frac{2}{-4} \rightarrow m=-\frac{1}{2}$

Falls to the Right (Rises to the left)

d.) Horizontal Line


$m=\frac{0}{5} \rightarrow m=0$

Horizontal Line (Have a slope $=0$)

e.) Vertical Line


$m=\frac{8}{0} \rightarrow m=$ undefine

Vertical Line (Has an Undefined slope) *Cannot divide by 0*

Example 3: Find the slope given two points or find the missing value.

a.) $(-1, 4)$; $(3, -2)$ ($x_1\ y_1$, $x_2\ y_2$)

$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{-2-4}{3--1}$ → $m=\frac{-2-4}{3+1}$ → $\frac{-6}{4}$ → $m=\frac{-3}{2}$

b.) $(3, 4)$; $(-3, 4)$ ($x_1\ y_1$, $x_2\ y_2$)

$m=\frac{4-4}{-3-3}$ Horizontal Line!

$m=\frac{0}{-6}$ $m=0$

c.) $(2, 5)$; $(4, 7)$ ($x_1\ y_1$, $x_2\ y_2$)

$m=\frac{7-5}{4-2}$

$m=\frac{2}{2}$ $m=1$

d.) $(-5, ½)$; $(-5, 3)$ ($x_1\ y_1$, $x_2\ y_2$)

$m=\frac{3-.5}{-5--5}$ $m=\frac{3-.5}{-5+5}$ → $m=\frac{2.5}{0}$ → m = undefine

e.) $(4, 3)$; $(x, 7)$ where $m = 2$ ($x_1\ y_1$, $x_2\ y_2$) $m=\frac{y_2-y_1}{x_2-x_1}$

$2=\frac{7-3}{x-4}$

$\frac{2}{1}=\frac{4}{x-4}$

$2(x-4)=4(1)$

$2x-8=4$

$+8\quad +8$

$\frac{2x}{2}=\frac{12}{2}$

$x=6$

f.) $(2, y)$; $(-6, 8)$ where $m = -½$ ($x_1\ y_1$, $x_2\ y_2$)

$-\frac{1}{2}=\frac{8-y}{-6-2}$

$\frac{-1}{2}=\frac{8-y}{-8}$

$2(8-y)=-8(-1)$

$16-2y=8$

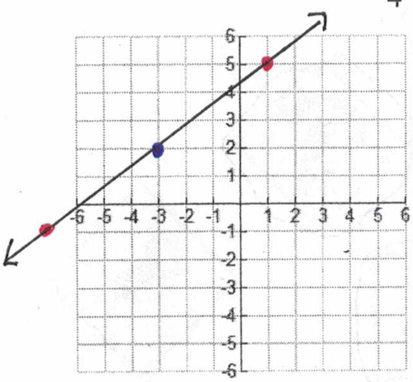
$-16\quad -16$

$\frac{-2y}{-2}=\frac{-8}{-2}$

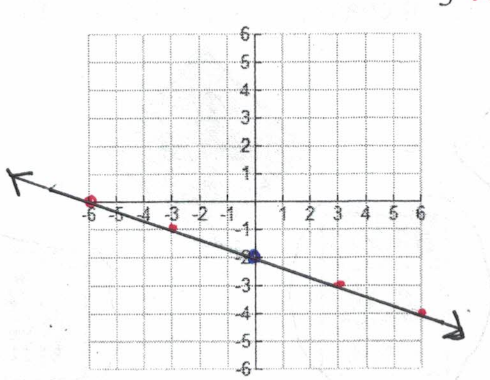
$y=4$

Example 4: Draw a line by plotting the given point and using the given slope to obtain a second point.

a.) $(-3, 2)$ with a slope $=\frac{3}{4}$ $\frac{\text{rise}}{\text{run}}$



b.) $(0, -2)$ with a slope $=-\frac{1}{3}$ $\frac{\text{rise}}{\text{run}}$



c.) $(4, 1)$ with a slope $=\phi$ (Undefin Vertical Line