4.3 – Exponential and Logarithmic Equations

Solving Exponential Equations Examples:

* ALWAYS ✓ YOUR SOLUTIONS *

Notes: 1.) Keep answers as (reduced) fractions when possible.
If a decimal can NOT be turned into a fraction – ROUND to 3 decimal places (thousandths)

Exponential Equations: Type 1 – Both Sides have the Same Base

1a.) $2^{2x+15} = 8$

$2^{2x+15} = 2^3$ ← SAME BASE

$2x+15 = 3$

$2x = -12$

$x = -6$

$\{-6\}$ ✓ ← Check Solution in Original Equation!

1b.) $4^{3x} = 32^{x+1}$

$(2^2)^{3x} = (2^5)^{x+1}$

$2^{6x} = 2^{5(x+1)}$

$6x = 5x + 5$

$x = 5$

$\{5\}$ ✓

1c.) $3^{2x+5} = \left(\frac{1}{9}\right)^{x-1}$

$3^{2x+5} = \left(\frac{1}{3^2}\right)^{x-1}$

$3^{2x+5} = 3^{-2(x-1)}$

$2x+5 = -2(x-1)$

$2x+5 = -2x+2$

$4x = -3$

$x = -3/4$

$\{-3/4\}$ ✓

1d.) $25 \cdot \left(\frac{1}{125}\right)^{x+4} = \sqrt{3125}$

$5^2 \cdot (5^{-3})^{x+4} = (5^5)^{1/2}$

$2 - 3(x+4) = 5(1/2)$

$2 - 3x - 12 = 5/2$

$-3x - 10 = 5/2$

$-3x = 25/2$

$x = -25/6$

$\{-25/6\}$ ✓

Exponential Equations: Type 2: Both Sides are NOT the Same Base – Take log (or ln) of both sides

1e.) $3^{2x-1} = 7$

$\log 3^{2x-1} = \log 7$

$(2x-1)\log 3 = \log 7$

$\frac{(2x-1)\log 3}{\log 3} = \frac{\log 7}{\log 3}$

$2x - 1 = \frac{\log 7}{\log 3}$

$2x - 1 = 1.771243749$

$+1 \quad +1$

$2x = 2.771243749$

$x \approx 1.386$

$\{1.386\}$ ✓

1f.) $3 \cdot 4^x + 11 = 2$

$-11 \quad -11$

$\frac{3 \cdot 4^x}{3} = \frac{-9}{3}$

$4^x = -3$

$\log 4^x = \log(-3)$

$x \log 4 = \log(-3)$

Cannot take the log of a negative #.

$\{\}$ ~ Empty Set

1g.) $\frac{2e^{5x-3}}{2} = \frac{16}{2}$

$e^{5x-3} = 8$

$\ln e^{5x-3} = \ln 8$

$5x - 3 = \ln 8$

$+3 \quad +3$

$\frac{5x}{5} = \frac{\ln(8)+3}{5}$

$x = \frac{(\ln(8)+3)}{5}$

$\{1.016\}$ ✓

1h.) $\frac{12}{1+e^{-x}} = \frac{2}{1}$ Proportion

$\frac{2(1+e^{-x})}{2} = \frac{12}{2}$

$1 + e^{-x} = 6$

$-1 \quad -1$

$e^{-x} = 5$

$\ln e^{-x} = \ln 5$

$-x = \ln 5$

$x = -\ln 5$

$x \approx -1.609$

$\{-1.609\}$ ✓