4.2 – Exponential and Logarithmic Functions

Exponential / Logarithmic Functions and Their Characteristics (Exp. and Log Functions are INVERSES)

– **exponential function** → a function in the form $y=(b)^x$ $y=a(b)^x$ where $b > 0$, $b \neq 1$, and x is IR.
* The variable is the exponent *

Exponential Function's Characteristics		Graphs of Exponential Functions	
Domain: $(-\infty,\infty)$	Range: $(0,\infty)$	a.) Graph of $y = (2)^x$	b.) Graph of $y = (½)^x$
Common Pt: $(0,1)$	Asymptote: HA $y=0$	Graph: $y = (2)^{x+\boxed{1}} + 2$	Graph: $y = (½)^{x-2} - 1$

Transforming Exp Graph: $y = b^{(x \pm c)} \pm d$

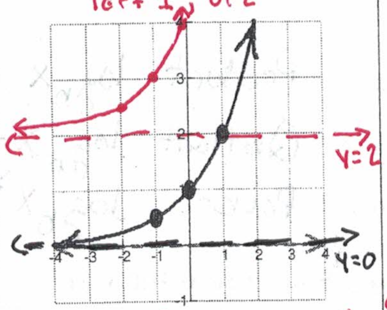
a.) # is on "outside" → + d: up – d: down

b.) # is on "inside" → + c: left – c: right

c.) Domain of Transform Graph: $(-\infty,\infty)$

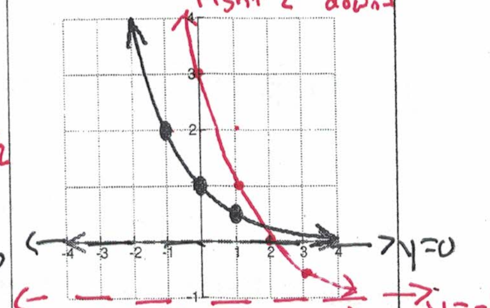
d.) Range of Transform Graph: (# of HA, ∞)

left 1, up 2



D: $(-\infty,\infty)$ R: $(2,\infty)$

right 2 down 1



D: $(-\infty,\infty)$ R: $(-1,\infty)$

* Cannot TAKE THE LOG of A NEGATIVE #!!

– **logarithmic function** → a function in the form $y=\log_b(x)$ or $\ln(x)$ where $b > 0$, $b \neq 1$, and ~~x is IR~~ y is IR. x > 0
← Natural Log

Logarithmic Function's Characteristics		Graphs of Logarithmic Functions	
Domain: $(0,\infty)$	Range: $(-\infty,\infty)$	a.) Graph of $y = \log_2(x)$	b.) Graph of $y = \log_{½}(x)$
Common Pt: $(1,0)$	Asymptote: VA $x=0$	Graph: $y = \log_2(x+1) - 3$	Graph: $y = \log_{½}(x-2) + 2$

Transforming Log Graph: $y = \log_b(x \pm c) \pm d$

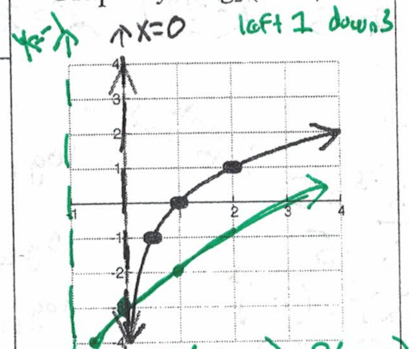
a.) # is on "outside" → + d: up – d: down

b.) # is on "inside" → + c: left – c: right

c.) Domain of Transform Graph: (# of VA, ∞)

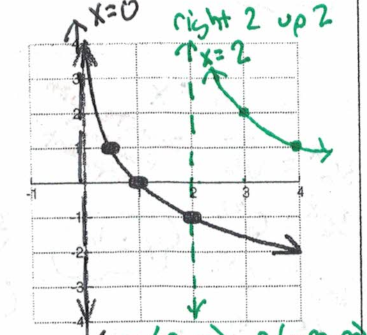
d.) Range of Transform Graph: $(-\infty,\infty)$

x=0 left 1 down 3



D$(-1,\infty)$ R$(-\infty,\infty)$

x=0 right 2 up 2 x=2



D$(2,\infty)$ R$(-\infty,\infty)$

Example 1: State the asymptote, domain, and range of each given function using interval notation.

	Given Exp / Log Function	Asymptote	Domain	Range
Exp	a.) $f(x) = 4^{x-3} + 5$ ← asymptote; right 3 up 5	HA $y=5$	$(-\infty,\infty)$	$(5,\infty)$
Log	b.) $f(x) = \log_3(x+4) - 3$ ← Asymptote; left 4 down 3	VA $x=-4$	$(-4,\infty)$	$(-\infty,\infty)$
Exp	c.) $f(x) = (⅓)^{x+5} - 2$ ← Asymptote; left 5 down 2	HA $y=-2$	$(-\infty,\infty)$	$(-2,\infty)$
Log	d.) $f(x) = \ln(x-4) + 1$; right 4 up 1	VA $x=4$	$(4,\infty)$	$(-\infty,\infty)$

$\log_3 9 \rightarrow \log_3 3^2 \rightarrow 2$

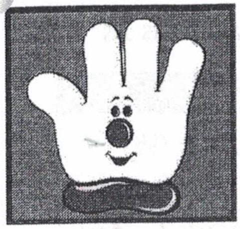
$e^x \approx 2.718$ natural base
* e is NOT a variable *

ln is the natural log

$\log_{10}$ is log

Properties of Logarithmic Functions

Basic Log Property (Hamburger Helper Hand) → helps to convert from LOG form to EXP FORM



Logarithmic Form		**Exponential Form**
$\log_b y = x$	⟷	$b^x = y$

Example 2: Convert

a.) $\log_2 8 = 3 \leftrightarrow$ $2^3 = 8$

b.) $\log_5 625 = 4 \leftrightarrow 5^4 = 625$

Laws of Logarithms →

Law # 1: Product $\log_b x + \log_b y \longleftrightarrow \log_b (xy)$

Law # 2: Quotient $\log_b x - \log_b y \longleftrightarrow \log_b (x/y)$

Law # 3: Power $\log_b x^y \longleftrightarrow y \log_b x$

Example 3: Evaluate each expression or find the value of x.

a.) $\log_3 9 = x$ Rewrite as Expo. $3^x = 9$ ← Make both sides have same base! $\cancel{3}^x = \cancel{3}^2$ $x = 2$	b.) $\log_4 8 = x$ $4^x = 8$ *Change both bases to 2! $(2^2)^x = 2^3$ $2^{2x} = 2^3$ $2x = 3$ $x = 3/2$	c.) $\log_2\left(\frac{1}{16}\right) = x$ $2^x = \frac{1}{16}$ ← change to base of 2! $2^x = \frac{1}{2^4}$ $\cancel{2}^x = \cancel{2}^{-4}$ $x = -4$	d.) $\log_8\left(\frac{1}{256}\right) = x$ $8^x = \frac{1}{256}$ $x = -8/3$ $(2^3)^x = \frac{1}{2^8}$ $2^{3x} = 2^{-8} \rightarrow 3x = -8$
e.) $\log_{36}\sqrt{6} = x$ rewrite with fractional exponent $36^x = \sqrt{6}$ $36^x = 6^{1/2}$ $x = 1/4$ $(6^2)^x = 6^{1/2}$ $\cancel{6}^{2x} = \cancel{6}^{1/2}$ $2x = \frac{1}{2}$	f.) $\log_x 5 = \frac{1}{3}$ rewrite with radical $x^{1/3} = 5$ $\sqrt[3]{x} = 5$ $(\sqrt[3]{x})^3 = (5)^3$ $x = 125$	g.) $\log(100)^4$ $\log_{10} 100^4$ $\log_{10}(10^2)^4$ $\log_{10}(10)^8$ 8	h.) $\ln\left(\frac{1}{e^3}\right)$ $\ln \cancel{e}^{-3}$ -3
i.) $\log_2 112 - \log_2 7$ rewrite as 1 log! $\log_2\left(\frac{112}{7}\right)$ $\log_2 16 = x$ $2^x = 16$ $x = 4$	j.) $\log_{12} 9 + \log_{12} 16$ $\log_{12}(9 \cdot 16)$ $\log_{12}(144) = x$ $12^x = 144$ $12^x = 12^2$ $x = 2$	k.) $e^{3\ln 2 - \ln 4}$ $e^{\ln 2^3 - \ln 4}$ $e^{\ln 8 - \ln 4}$ $e^{\ln 8/4}$ $\cancel{e}^{\cancel{\ln} 2}$ 2	l.) $\log_{10}\sqrt{\frac{1}{10}} = x$ $10^x = \sqrt{1/10}$ $10^x = (1/10)^{1/2}$ $\cancel{10}^x = (\cancel{10}^{-1})^{1/2}$

$2^x = 2^4$

$x = -1/2$

index, radical, radical

$\sqrt{x} = x^{1/2}$

$\sqrt[5]{x^2} = x^{2/5}$

$x^{-5} \rightarrow \frac{1}{x^5}$

$\frac{1}{y^3} \rightarrow y^{-3}$

- Sometimes it is easier to convert to exponential!
- If in exponential form, you want both sides to have the SAME BASE!