3.3 – Polynomial Functions and Their Graphs

Polynomial Function is a function in the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$ where the degree "n" should be the highest exponent (b/c it is in Standard Form) and it's graph is Smooth and continuous.

ie. $P(x) = 5x^4 - 3x^3 + x^2 - 2x + 3$: degree = 4.

Leading Coefficient — Standard Form

Characteristics of Polynomial Functions

- **End Behavior** → a description of the y-values as x becomes large in the positive and small in the negative directions; "Approaching" both $+\infty$ and $-\infty$.

- End Behavior Notation: $x \to -\infty,\ y \to ?$ means What is "y" doing on the LEFT SIDE?

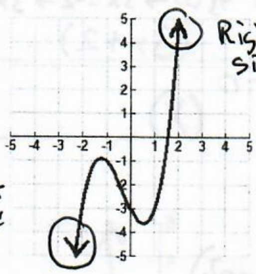
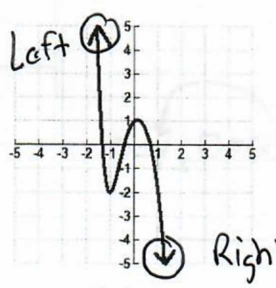
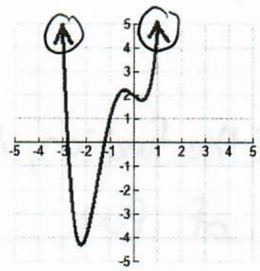
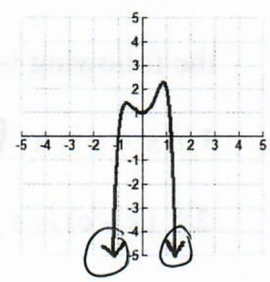
  $x \to \infty,\ y \to ?$ means What is "y" doing on the RIGHT SIDE?

The symbol ⟶ means "Approaching".

When you write "END BEHAVIOR" you need this entire box.

End Behavior can be determined in 2 ways: 1.) By looking at the function's graph.

2.) By interperting the function's degree and "LC" (Leading Coefficient)

End Behavior of "ODD" Degree Graphs		**End Behavior of "Even" Degree Graphs**	
$P(x) = x^3 + x^2 - 2x - 3$	$P(x) = -x^5 + x^4 - 4x^2 + x + 1$	$P(x) = x^4 + 3x^3 - x + 2$	$P(x) = -2x^6 + x^3 + 2x^2 + 1$
Degree = 3 (ODD) LC = 1 (Positive)	Degree = 5 (ODD) LC = -1 (negative)	Degree = 4 (even) LC = 1 (positive)	Degree = 6 (even) LC = -2 (negative)
 Right side; Left side	 Left; Right		
$x \xrightarrow{L} -\infty,\ y \to -\infty$	$x \xrightarrow{L} -\infty,\ y \to \infty$	$x \xrightarrow{L} -\infty,\ y \to \infty$	$x \xrightarrow{L} -\infty,\ y \to -\infty$
$x \xrightarrow{R} \infty,\ y \to \infty$	$x \xrightarrow{R} \infty,\ y \to -\infty$	$x \xrightarrow{R} \infty,\ y \to \infty$	$x \xrightarrow{R} \infty,\ y \to -\infty$

Conclusion about end behavior: THE "ENDS" go in Opposite directions.

ODD Degree w/ + LC Left $-\infty$, Right ∞

ODD Degree w/ − LC Left ∞, Right $-\infty$

Conclusion about end behavior: The "ENDS" go in SAME direction.

Even Degree w/ + LC both sides ∞

Even Degree w/ − LC both sides $-\infty$

Example 1: Identify the degree and leading coefficient and describe the end behavior.

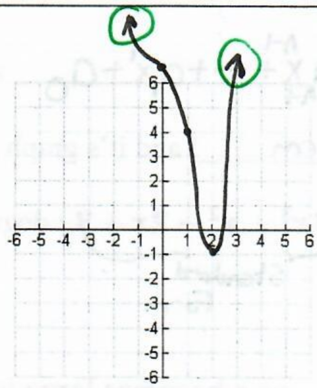
a.) $f(x) = 2x^{4} - 5x^3 + 7$

Degree: 4 LC: +2

End Behavior:

$x \to -\infty$, $y \to$ ∞

$x \to \infty$, $y \to$ ∞



b.) $g(x) = (x-2)^{1}(x+3)^{2}$ FACTORED FORM!

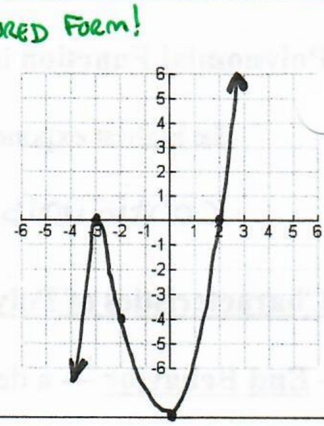
Degree: 1+2 → 3 LC: 1

$(1)^1(1)^2$

End Behavior:

$x \to -\infty$, $y \to$ $-\infty$

$x \to \infty$, $y \to$ ∞



c.) $h(x) = -2(x+3)^{4}(x-2)^{2}$ ←Factored Form

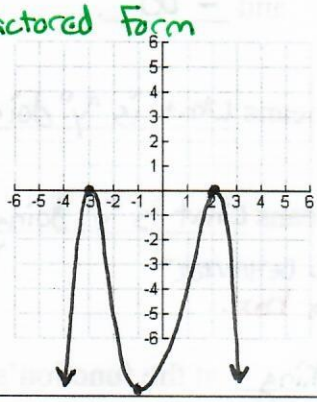
Degree: 4+2 → 6 LC: −2

Add the exponents $-2(1)^4(1)^2$

End Behavior:

$x \to -\infty$, $y \to$ $-\infty$

$x \to \infty$, $y \to$ $-\infty$



d.) $f(x) = 2x^4 + 8x^2 - 3x^5$ ←NOT IN STANDARD FORM

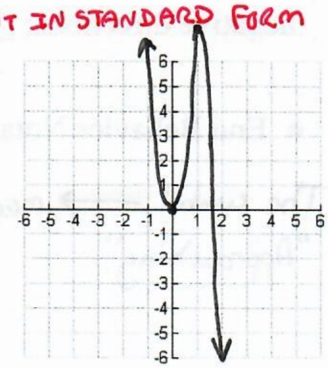
$= -3x^5 + 2x^4 + 8x^2$

Degree: 5 LC: −3

End Behavior:

$x \to -\infty$, $y \to$ ∞

$x \to \infty$, $y \to$ $-\infty$



- **Zero(s) (of a polynomial function)** → has the following definitions:

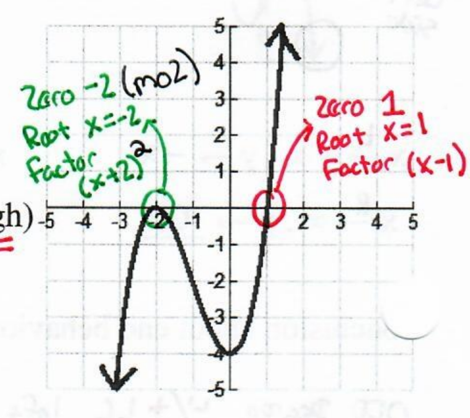
- **Graphically** a Point (x,y) on the graph where it crosses the x-axis; x-intercept → (x,0)
- **Algebraically** it is the ROOT x = # of a function.
- If " c " is a zero of a polynomial function P (x), then the following is true:

  1.) x = c is a ROOT of the function, where P(x)=0; y=0

  2.) (x – c) is a Factor of P(x)

- Degree of polynomial = the total # of zeros of a polynomial function. Real and Complex (i)
- A root of P(x) has a multiplicity of 1 (counts only once) if its root CROSSES (through) the x-axis.
- A root of P (x) has a multiplicity of 2 if its root only touches (at) the x-axis.

REAL zero (x-intercept)	factor Unsolve for "x"
3	x=3 → x−3=0 → (x−3)
−1	x=−1 → x+1=0 → (x+1)
½	(2) x=½ (2) → 2x=1 → 2x−1=0 (2x−1)
−⅔	(3) x=−⅔ (3) → 3x=−2 → 3x+2=0 (3x+2)
0	x=0 (x)



You can use a graphing calculator find the **REAL ZEROS** of a polynomial function.

Step 1: y_1 = POLYNOMIAL FUNCTION
Step 2: $y_2 = 0$
Step 3: Graph
Step 4: 2nd Trace #5:Intersect – Enter x3
Step 5: Repeat process for each zero (x-intercept) for the function

Example 2: Find the zeros and the factors for each polynomial P(x) using a graphing calculator.

	2 zeros a.) $P(x)=3x^2-x-2$		3 zeros b.) $P(x)=4x^3+5x^2-23x-6$			4 zeros c.) $P(x)=2x^4+11x^3-8x^2-80x$			
Zeros	$-2/3$	1	-3	$-1/4$	2	-4	-4	0	$2.5 \to 5/2$
Factors	$x=-2/3$ $3x=-2$ $3x+2=0$ $(3x+2)$	$x=1$ $x-1=0$ $(x-1)$	$x=-3$ $x+3=0$ $(x+3)$	$x=-1/4$ $4x=-1$ $4x+1=0$ $(4x+1)$	$x=2$ $x-2=0$ $(x-2)$	multiplicity of 2 $x=-4$ $x+4=0$ $(x+4)^2$		$x=0$ (x)	$x=\frac{5}{2}$ $2x=5$ $2x-5=0$ $(2x-5)$
Factored Form	$P(x)=(3x+2)(x-1)$		$P(x)=(x+3)(4x+1)(x-2)$			$P(x)=(x+4)^2(x)(2x-5)$			

If given the zeros of a polynomial function, you can work backwards to find the Original Polynomial by converting the zeros to factors and then multiplying the factors. Make sure you write ***P(x)=*** in front of the function.

Example 3: Find the polynomial P (x) with the given zeros.

a.) zeros = -4, 3 ← degree of 2

Roots → $x=-4$ $x=3$
↓
Factors $(x+4)\cdot(x-3)$

$P(x)=(x+4)(x-3)$

$x^2-3x+4x-12$

$P(x)=x^2+x-12$

b.) zeros = -2, 0, ¼ ← degree of 3

Roots → $x=-2$, $x=0$, $x=1/4$
↓
Factors $(x+2)\cdot(x)\cdot(4x-1)$

$P(x)=x(x+2)(4x-1)$

$(x^2+2x)(4x-1)$

$4x^3-x^2+8x^2-2x$

$P(x)=4x^3+7x^2-2x$

c.) zeros = – 2 (mult of 2) , 4 ← degree of 3
← counts twice

Roots → $x=-2$, $x=-2$, $x=4$
↓
Factors $(x+2)(x+2)(x-4)$

$(x+2)(x+2)$
↓
$x^2+2x+2x+4$
↓
x^2+4x+4

	x^2	$+4x$	$+4$
x	x^3	$4x^2$	$4x$
-4	$-4x^2$	$-16x$	-16

$P(x)=x^3-12x-16$

d.) zeros = – 3 , – ½ , ¾ (mult of 2) ← degree of 4

$x=-3 \to (x+3)$

$x=-\frac{1}{2} \to (2x+1)$

$x=\frac{3}{4} \to (4x-3)^2$

$x=3/4$

	x	$+3$
$2x$	$2x^2$	$6x$
$+1$	x	3

$2x^2+7x+3$

	$4x$	-3
$4x$	$16x^2$	$-12x$
-3	$-12x$	$+9$

$16x^2-24x+9$

	$2x^2$	$+7x$	$+3$
$16x^2$	$32x^4$	$112x^3$	$48x^2$
$-24x$	$-48x^3$	$-168x^2$	$-72x$
$+9$	$18x^2$	$63x$	27

$P(x)=32x^4+64x^3-102x^2-9x+27$

- **Local Extrema** → are critical points, "turning points", of a graph and are the minimum (lowest) and Maximum (highest) point**S**.
  - If a polynomial function P (x) has a degree of "n" then there are at most (n − 1) local extrema (turning points).

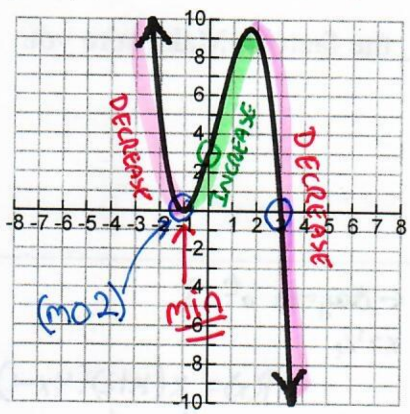
- **Critical Point (CP)** → point on a graph where the graph changes direction.
  - **Increasing**: state the **CP's** x-value when the y-values are going up.
  - **Decreasing**: state the **CP's** x-value when the y-values are going down.

*Use only the x-values of the turning points (max/min) and write in Interval Notation.

***Write your INTERVALS of Increasing and Decreasing in **INTERVAL NOTATION**!

Example 4: Complete all the blank information about each polynomial P (x).

a.) $P(x) = -x^3 + x^2 + 5x + 3$ ← degree



zeros: −1 (mo 2), 3

factors: $(x+1)^2$ $(x-3)$

extrema: min (−1,0) max (1.7, 9.5)

y-int: (0,3)

End Behavior: $x \xrightarrow{L} -\infty$, $y \to \infty$

$x \xrightarrow[R]{} \infty$, $y \to -\infty$

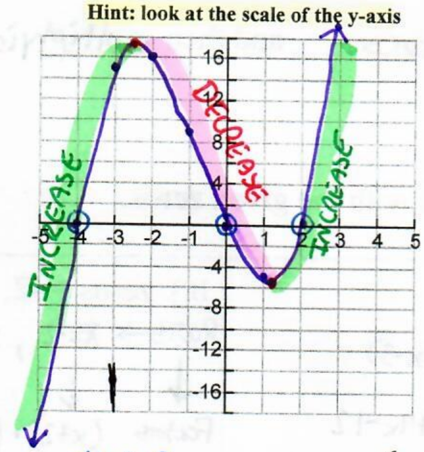
Interval Increase: (−1, 1.7)

Use the x-values of the max/min.

Interval Decrease: (−∞, −1) ∪ (1.7, ∞)

b.) $P(x) = x^3 + 2x^2 - 8x$

Hint: look at the scale of the y-axis



zeros: −4, 0, 2

factors: (x+4) (x−2), (x)

extrema: min (1.1, −5) max (−2.4, 16.9)

y-int: (0,0)

End Behavior: $x \xrightarrow{L} -\infty$, $y \to -\infty$

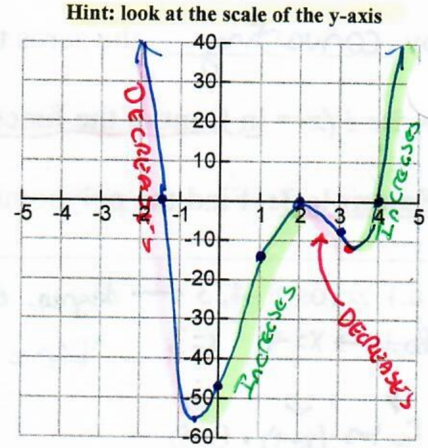
$x \xrightarrow[R]{} \infty$, $y \to \infty$

Interval Increase: (−∞, −2.4) ∪ (1.1, ∞)

Interval Decrease: (−2.4, 1.1)

c.) $P(x) = 2x^4 - 13x^3 + 16x^2 + 28x - 48$

Hint: look at the scale of the y-axis



zeros: −3/2, 2 (mo 2), 4

factors: (2x+3) $(x-2)^2$ (x−4)

extrema: min (−.5, −56.3) (3.4, −11.5) max (2,0)

y-int: (0, −48)

End Behavior: $x \xrightarrow{L} -\infty$, $y \to \infty$

$x \xrightarrow[R]{} \infty$, $y \to \infty$

Interval Increase: (−.5, 2) ∪ (3.4, ∞)

Interval Decrease: (−∞, −.5) ∪ (2, 3.4)