3.2 – Dividing Polynomials: Monomial vs Long vs. Synthetic Division

Division of an expression can be represented (written) three different ways:

a.) **Fraction Bar**

$$\frac{x^2+5x+6}{x+2}$$ ← Dividend

↑ Divisor

b.) **Raised to Power of -1**

$$(x^2+5x+6)(x+2)^{-1}$$

$$\to (x^2+5x+6)\left(\frac{1}{x+2}\right) \to \frac{x^2+5x+6}{x+2}$$

c.) **Division Sign**

$$(x^2+5x+6) \div (x+2)$$

Division of Polynomials is away to find ALL of the zeros (factors) roots, and x-intercepts of a polynomial.

- If there is a Remainder, then the divisor (factor) is NOT a zero of the polynomial!
- If there is no Remainder, then the divisor (factor) IS a zero of the polynomial!

– **Monomial Division** → a way to divide a polynomial when you only have a monomial (1-term).

Take the divisor (monomial) and put it underneath ALL terms of the dividend (polynomial). Reduce all coefficients and use the Division of Exponent rule.

Example 1: Divide by a monomial.

a.) $(15a^6b^4 + 10a^4b^3 - 5a^2b^2) \div 5ab$ (division!) $\frac{15a^6b^4}{5ab} + \frac{10a^4b^3}{5ab} - \frac{5a^2b^2}{5ab}$ $3a^{6-1}b^{4-1} + 2a^{4-1}b^{3-1} - 1a^{2-1}b^{2-1}$ $3a^5b^3 + 2a^3b^2 - ab$	b.) $(8x^5 + 16x^3 - 2x^2 + 4x)\boxed{(4x^2)^{-1}}$ (division) $\frac{8x^5}{4x^2} + \frac{16x^3}{4x^2} - \frac{2x^2}{4x^2} + \frac{4x}{4x^2}$ ← $1x^{-1}$ cannot leave w/negative exponent. $2x^3 + 4x - \frac{1}{2} + \frac{1}{x}$

* 8th degree trinomial *   * Cubic 4-term polynomial

Quick Review of Long Division With Numbers

Step 1 – Divide (number of times divisor go into…)

Step 2 – Multiply (product of step 1 and divisor)

Step 3 – Subtract (line up and find difference)

Step 4 – Bring Down (repeat and remainder / divisor)

a.) divisor → 2, Quotient → 65, 130 ← dividend

$$\begin{array}{r} 65 \\ 2\overline{)130} \\ -12\downarrow \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

65

b.)

$$\begin{array}{r} 86 \\ 5\overline{)432} \\ -40\downarrow \\ \hline 32 \\ -30 \\ \hline 2 \end{array}$$

↑ remainder

$86\frac{2}{5}$

– **Long Division** → a way to divide a polynomial if the divisor is in the form $ax \pm c;\ a > 1$ (e.g. $2x+5$) OR $1x \pm c$ and this process is similar to how you divide numbers like in elementary school.

****MUST USE THIS PROCESS WHEN $a > 1$ IN THE DIVISOR.****

Example 2: Divide using long division.

a.) $\underbrace{(x^2+2x-24)}_{\text{Dividend}} \div \underbrace{(x-4)}_{\text{Divisor}}$

$$\begin{array}{r} x + 6 \\ x-4\,\overline{)\,x^2+2x-24} \\ -x^2+4x \quad\;\; \\ \hline 6x-24 \\ -6x+24 \\ \hline 0 \end{array}$$

ax

$x+6$

Here are some "rules" to follow:

1) Polynomial should be in Standard Form.
2) If missing a degree (power) use "0" as a place holder.
3) Ask yourself, "What times "ax" gives you "ax^2"? * Powers will align vertically! *
4) multiply and the subtract (change signs).
5) Repeat the process; Steps 3-4 until no more division is possible.

b.) $(x^3+13x^2-12x-8)(x+2)^{-1}$ ← division

$$\begin{array}{r} x^2+11x-34 \\ x+2\,\overline{)\,x^3+13x^2-12x-8} \\ -x^3+2x^2 \qquad\qquad\;\; \\ \hline 11x^2-12x \quad\; \\ -11x^2+22x \quad\; \\ \hline -34x-8 \\ +34x+68 \\ \hline 60 \end{array}$$

60 remainder

$x^2+11x-34+\dfrac{60}{x+2}$

c.) $\dfrac{8x^4-4x^2+x-2}{2x+1}$

*missing a degree of 3 use $0x^3$ as a place holder!

$$\begin{array}{r} 4x^3-2x^2-x+1 \\ 2x+1\,\overline{)\,8x^4+0x^3-4x^2+x-2} \\ -8x^4+4x^3 \qquad\qquad\qquad \\ \hline -4x^3-4x^2 \qquad\quad \\ +4x^3+2x^2 \qquad\quad \\ \hline -2x^2+x \quad\; \\ +2x^2+x \quad\; \\ \hline 2x-2 \\ -2x+1 \\ \hline -3 \end{array}$$

$4x^3-2x^2-x+1-\dfrac{3}{2x+1}$

– **Synthetic Division** → a shortcut way to divide a polynomial if the divisor is ONLY in the form $(1x \pm c)$ $(x+2)$ $a=1$

CAN ONLY USE THIS PROCESS WHEN $a=1$ IN THE DIVISOR.

Here are some "rules" to follow:

1. Take the Opposite of "c" which goes in the half box (upper left corner) ← located in the divisor!
2. Write the coefficients of the polynomial in descending order of Powers!
3. If you are missing any powers, then put a zero for that position.
4. Bring down the 1st coefficient (use an arrow)
5. Multiply the leading coefficient by the value in the box and place the product under the next coefficient.
6. ADD the numbers and **repeat the process** until no numbers are left.
7. Answer (quotient) will need to be written as a polynomial that is ***one degree less than what you started with***.
8. Remainder (if any) will need to be written with the quotient.

Synthetic Division SET-UP:

$(4x^4 - 2x^2 + 5x - 1) \div (x + 3)$

Is dividend in Standard Form? Yes!

$x+3=0$
$x=-3$ ← 1

$4x^4 \quad 0x^3 \quad -2x^2 \quad 5x \quad -1$

1 -3 | 4 0 -2 5 -1 ← 2 and 3

4 ↓

4

-3 | 4 0 -2 5 -1

↓ + -12 6

5 → 4 -12

Example 3: **Divide using synthetic division.** * remainder → $\frac{\text{remainder}}{\text{divisor}}$

a.) $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$	b.) $(3x^5 + 2x^4 - x^2 + 4x) \div (x + 1)$	c.) $(20x + 2 - 21x^2 + x^4) \div (5 + x)$
Dividend, divisor	Is the dividend in Standard Form? YES	Standard Form? NO
Is dividend in Standard Form? Yes	$x+1=0$, $x=-1$	missing degrees? Yes
$x-2=0$, $x=2$	*Missing degree of 3, use 0 as placeholder.	$(x^4+0x^3-21x^2+20x+2) \div (x+5)$
		$x+5=0$, $x=-5$
2 | 5 -13 10 -8	-1 | 3 2 0 -1 4 0	-5 | 1 0 -21 20 2
↓ +10 +(-6) +(8)	↓ -3 1 -1 2 -6	↓ +(-5) +25 +(-20) 0
5 -3 4 | 0	3 -1 1 -2 6 | -6	1 -5 4 0 | 2
Remainder box	7 $3x^4 - x^3 + x^2 - 2x + 6 - \frac{6}{x+1}$	$x^3 - 5x^2 + 4x + \frac{2}{x+5}$
The new polynomial is 1 degree less than the original.	8 remainder!	remainder!
$5x^2 - 3x + 4$		

Example 4: Use either long or synthetic division to complete each geometry problem.

a.) The area of a rectangle is $2x^2 - 11x + 15$ square meters. The width of the rectangle is $x - 3$. What is the length?

$A = lw$

$$\left.\begin{aligned} A &= (2x^2-11x+15)m^2 \\ l &= ? \\ w &= (x-3) \end{aligned}\right\} l = \frac{A}{w}$$

$(2x^2-11x+15) \div (x-3)$ $a = 1$

* Synthetic ÷ *

$$\begin{array}{r|rrr} 3 & 2 & -11 & 15 \\ & \downarrow & +6 & +(-15) \\ \hline & 2 & -5 & 0 \end{array}$$

$2x - 5$

The length is $(2x-5)$ meters.

b.) A rectangle has an area of $4x^3 + 10x^2 - x + 15$ square feet. The length of the rectangle is $4x^2-2x+5$ feet. What is the width?

$A = lw$

$$\left.\begin{aligned} A &= 4x^3+10x^2-x+15 \\ l &= 4x^2-2x+5 \\ w &= ? \end{aligned}\right\} w = \frac{A}{l}$$

$(4x^3+10x^2-x+15) \div (4x^2-2x+5)$

* Long division *

$$\begin{array}{r} x+3 \\ 4x^2-2x+5 \,\big)\overline{\,4x^3+10x^2-x+15} \\ -4x^3+2x^2-5x \quad\downarrow \\ \hline 12x^2-6x+15 \\ -12x^2+6x-15 \\ \hline 0 \end{array}$$

The width is $(x+3)$ feet.

c.) The area of a triangle is $15x^4 - 17x^3 + 13x^2 - 21x + 6$. The length of the base is $5x^3-4x^2+3x-6$. What is the height?

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$A = 15x^4-17x^3+13x^2-21x+6$

$b = 5x^3-4x^2+3x-6$

$h = ?$

$h = \frac{2A}{b}$

$$h = \frac{2(15x^4-17x^3+13x^2-21x+6)}{5x^3-4x^2+3x-6}$$

$$h = \frac{30x^4-34x^3+26x^2-42x+12}{5x^3-4x^2+3x-6}$$

* LONG DIVISION *

$$\begin{array}{r} 6x-2 \\ 5x^3-4x^2+3x-6 \,\big)\overline{\,30x^4-34x^3+26x^2-42x+12} \\ -30x^4+24x^3-18x^2+36x \quad\downarrow \\ \hline -10x^3+8x^2-6x+12 \\ +10x^2-8x^2+6x-12 \\ \hline 0 \end{array}$$

The width is $(6x-2)$ units