

(FRACTIONS)

- **radical equation** → an equation that has a variable in the Radicand or a variable with a Rational Exponent

- **square root equation** – a radical equation where index = 2 or an expression is raised to 1/2 ← Square root
- **extraneous solution** – a type of solution that produces a FALSE solution when CHECKED back into the Original Equation.

Steps to Solving a RADICAL (Sq Root) EQUATION:

Step 1 → a.) If have ONE RADICAL, then get the radical (sq root) BY ITSELF.
 b.) If have TWO RADICALS, then get radical (sq root) ON EITHER SIDE of equal sign.

Step 2 → Raise EACH SIDE to a POWER that's = INDEX number (For sq roots, square each side)
 May have to use the Box Method (depends on what is given in the problem).

Step 3 → You will then have to solve for the variable using algebraic methods already learned...
 a.) If there is NO Square Term (x^2) then solve the LINEAR EQUATION (isolate variable).
 b.) If there is a Square Term (x^2) then solve the QUADRATIC EQUATION (get eq = 0).

Step 4 → When you are TOTALLY FINISHED, you **MUST CHECK YOUR SOLUTIONS!!!!**
 There may be extraneous solution(s) which are NOT part of the FINAL ANSWER!
 It could happen where ALL the solutions do NOT work where answer is no solution! No Solution or \emptyset or $\{ \}$

Examples: Solve each radical (sq root) equation. Make sure to check for extraneous solution(s).

1.) $\sqrt{4-12x} + 8 = 16$
 $\sqrt{4-12x} = 8$
 $(\sqrt{4-12x})^2 = (8)^2$
 $4-12x = 64$
 $-12x = 60$
 $x = -5$

CHECK SOLUTION(S)
 $\sqrt{4-12x} + 8 = 16$
 $\sqrt{4-12(-5)} + 8 = 16$
 $\sqrt{64} + 8 = 16$
 $8 + 8 = 16$
 $16 = 16$ ✓ True

2.) $(-6-2x)^{1/2} = (3x+14)^{1/2}$
 $\sqrt{-6-2x} = \sqrt{3x+14}$
 $(\sqrt{-6-2x})^2 = (\sqrt{3x+14})^2$
 $-6-2x = 3x+14$
 $-5x = 20$
 $x = -4$

CHECK SOLUTION(S)
 $\sqrt{-6-2x} = \sqrt{3x+14}$
 $\sqrt{-6-2(-4)} = \sqrt{3(-4)+14}$
 $\sqrt{2} = \sqrt{2}$ ✓ True

3.) $x = \sqrt{-1-2x}$
 $(x)^2 = (\sqrt{-1-2x})^2$
 $x^2 = -1-2x$
 $x^2 + 2x + 1 = 0$ FACTOR!
 $(x+1)(x+1) = 0$
 $x+1 = 0$
 $x = -1$ extraneous solution

CHECK SOLUTION(S)
 $x = \sqrt{-1-2x}$
 $(-1) = \sqrt{-1-2(-1)}$
 $-1 = \sqrt{1}$
 $-1 \neq 1$ FALSE
 * Even index cannot give a negative solution!

4.) $x+2 = \sqrt{7x+8}$
 $(x+2)^2 = (\sqrt{7x+8})^2$
 $x^2 + 4x + 4 = 7x + 8$
 $x^2 - 3x - 4 = 0$ FACTOR
 $(x+1)(x-4) = 0$
 $x+1 = 0$ or $x-4 = 0$
 $x = -1$ or $x = 4$

CHECK SOLUTION(S)
 $x+2 = \sqrt{7x+8}$
 $(-1)+2 = \sqrt{7(-1)+8}$
 $1 = \sqrt{1}$
 $1 = 1$ ✓ True
 $x+2 = \sqrt{7x+8}$
 $4+2 = \sqrt{7(4)+8}$
 $6 = \sqrt{36}$
 $6 = 6$ ✓ True

5.) $\sqrt{12-2x} + 6 = x$
 $\sqrt{12-2x} = x-6$
 $(\sqrt{12-2x})^2 = (x-6)^2$ Box or FOIL
 $12-2x = x^2 - 12x + 36$
 $0 = x^2 - 10x + 24$
 $0 = (x-6)(x-4)$
 $x = 6$ or $x = 4$
 extraneous solution

CHECK SOLUTION(S)
 $\sqrt{12-2x} + 6 = x$
 $\sqrt{12-2(6)} + 6 = 6$ ✓ True
 $\sqrt{0} + 6 = 6 \rightarrow 6 = 6$
 $\sqrt{12-2x} + 6 = x$
 $\sqrt{12-2(4)} + 6 = 4$ FALSE
 $\sqrt{4} + 6 = 4 \rightarrow 8 \neq 4$
 $2+6 = 4$

6.) $2(x-3)^{1/2} = \frac{1}{3}$
 $\frac{2}{(x-3)^{1/2}} = \frac{1}{3}$
 $\frac{2}{\sqrt{x-3}} = \frac{1}{3}$
 $\sqrt{x-3} = 6$
 $(\sqrt{x-3})^2 = (6)^2$
 $x-3 = 36$
 $x = 39$

CHECK SOLUTION(S)
 $\frac{2}{\sqrt{x-3}} = \frac{1}{3}$
 $\frac{2}{\sqrt{39-3}} = \frac{1}{3}$
 $\frac{2}{\sqrt{36}} = \frac{1}{3}$
 $\frac{2}{6} = \frac{1}{3} \rightarrow \frac{1}{3} = \frac{1}{3}$ ✓ True