3.2 – Dividing Polynomials: Monomial vs Long vs. Synthetic Division

Division of an expression can be represented (written) three different ways:

a.) **Fraction Bar**

$\frac{x^2+5x+6}{x+2}$ ← Dividend

↑ Divisor

b.) **Raised to Power of -1**

$(x^2+5x+6)(x+2)^{-1}$

$\rightarrow (x^2+5x+6)\left(\frac{1}{x+2}\right) \rightarrow \frac{x^2+5x+6}{x+2}$

c.) **Division Sign**

$(x^2+5x+6) \div (x+2)$

Division of Polynomials is away to find ALL of the zeros (factors), roots, and x-intercepts of a polynomial.

- If there is a Remainder, then the divisor (factor) is NOT a zero of the polynomial!
- If there is no Remainder, then the divisor (factor) IS a zero of the polynomial!

– **Monomial Division** → a way to divide a polynomial when you only have a monomial (1-term).

Take the divisor (monomial) and put it underneath ALL terms of the dividend (polynomial). Reduce all coefficients and use the Division of Exponent rule.

Example 1: Divide by a monomial.

a.) $(15a^6b^4 + 10a^4b^3 - 5a^2b^2) \div 5ab$ (division!)	b.) $(8x^5 + 16x^3 - 2x^2 + 4x)\boxed{(4x^2)^{-1}}$ (division)
$\frac{15a^6b^4}{5ab} + \frac{10a^4b^3}{5ab} - \frac{5a^2b^2}{5ab}$	$\frac{8x^5}{4x^2} + \frac{16x^3}{4x^2} - \frac{2x^2}{4x^2} + \frac{4x}{4x^2}$ ← $1x^{-1}$ cannot leave w/negative exponent.
$3a^{6-1}b^{4-1} + 2a^{4-1}b^{3-1} - 1a^{2-1}b^{2-1}$	$2x^3 + 4x - \frac{1}{2} + \frac{1}{x}$
$3a^5b^3 + 2a^3b^2 - ab$	
* 8th degree trinomial *	* Cubic 4-term polynomial

Quick Review of Long Division With Numbers

Step 1 – Divide (number of times divisor go into…)

Step 2 – Multiply (product of step 1 and divisor)

Step 3 – Subtract (line up and find difference)

Step 4 – Bring Down (repeat and remainder / divisor)

a.) divisor, Quotient

$$\begin{array}{r} 65 \\ 2\overline{)130} \\ -12\downarrow \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

← dividend

65

b.)

$$\begin{array}{r} 86 \\ 5\overline{)432} \\ -40\downarrow \\ \hline 32 \\ -30 \\ \hline 2 \end{array}$$

↖ remainder

$86\frac{2}{5}$

– **Long Division** → a way to divide a polynomial if the divisor is in the form $\overset{2x+5}{ax \pm c}$; $a > 1$ OR $1x \pm c$ and this process is similar to how you divide numbers like in elementary school.

****MUST USE THIS PROCESS WHEN $a > 1$ IN THE DIVISOR.****

Example 2: Divide using long division.

a.) $\underbrace{(x^2+2x-24)}_{\text{Dividend}} \div \underbrace{(x-4)}_{\text{Divisor}}$ ax $x+6$ $x-4 \overline{)\, x^2+2x-24}$ $-x^2+4x$ $6x-24$ $-6x+24$ 0 $x+6$	Here are some "rules" to follow: 1) Polynomial should be in Standard Form. 2) If missing a degree (power) use "0" as a place holder. 3) Ask yourself, "What times "ax" gives you "ax^2"? * Powers will align vertically! * 4) multiply and the subtract (change signs). 5) Repeat the process; Steps 3-4 until no more division is possible.
b.) $(x^3+13x^2-12x-8)(x+2)^{-1}$ ← division $x^2+11x-34$ $x+2 \overline{)\, x^3+13x^2-12x-8}$ $-x^3+2x^2$ $11x^2-12x$ $-11x^2+22x$ $-34x-8$ $+34x+68$ 60 remainder $x^2+11x-34+\frac{60}{x+2}$	c.) $\frac{8x^4-4x^2+x-2}{2x+1}$ *missing a degree of 3 use $0x^3$ as a place holder! $4x^3-2x^2-x+1$ $2x+1 \overline{)\, 8x^4+0x^3-4x^2+x-2}$ $-8x^4+4x^3$ $-4x^3-4x^2$ $+4x^3+2x^2$ $-2x^2+x$ $+2x^2+x$ $2x-2$ $-2x+1$ -3 $4x^3-2x^2-x+1-\frac{3}{2x+1}$