

- **rational exponent** → a way to rewrite a radical using **FRACTIONAL exponents.**

Rational Exponent Property → If have $\sqrt[n]{x^m}$, then $x^{m/n}$ $\frac{m}{n}$ (exponent of radicand)
 $\frac{m}{n}$ (index #)

Ex: $\sqrt[3]{2} = 2^{1/3}$ $\sqrt{5} = 5^{1/2}$ $\sqrt[4]{x^5} = x^{5/4}$

** All the same exponent properties STILL apply to rational exponents: $x^{3/7} \rightarrow \sqrt[7]{x^3}$

Zero Power	Multiplying Like Bases	Dividing Like Bases	Power to a Power
$a^0 = 1$	$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{m \cdot n}$
Negative Exp in Num	Negative Exp in Den	Product to a Power	Quotient to a Power
$a^{-n} = \frac{1}{a^n}$	$\frac{1}{a^{-n}} = a^n$	$(ab)^n = a^n b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 1: Simplify using rational exponents. Keep your answers in rational exponent form.

a.) $3m^{\frac{4}{3}} \cdot 2m^{\frac{1}{2}}$

$(3 \cdot 2)m^{\frac{4}{3} + \frac{1}{2}}$

$(6m^{\frac{5}{6}})$

$\frac{4}{3} = \frac{8}{6}$
 $+\frac{1}{2} = \frac{3}{6}$
 $\frac{5}{6}$

b.) $\left(\frac{x^{\frac{1}{4}}}{y^{\frac{3}{4}}}\right)^{12}$

$\frac{(x^{\frac{1}{4}})^{12}}{(y^{\frac{3}{4}})^{12}} \rightarrow \frac{x^3}{y^9}$

$\frac{x^3}{y^9}$

c.) $\frac{3x^{\frac{2}{5}} \cdot 4x^{\frac{4}{5}}}{2x^{\frac{1}{3}}}$ Simplify the top first!

$\frac{(3 \cdot 4)x^{-2/5 + 4/5}}{2x^{1/3}} \rightarrow \frac{12x^{2/5}}{2x^{1/3}}$

$\rightarrow 6x^{2/5 - 1/3} \rightarrow 6x^{1/15}$

d.) $\left(16x^{\frac{2}{3}}\right)^{\frac{3}{4}}$ $\sqrt[4]{16^3}$

$16^{3/4} \cdot (x^{-2/3})^{3/4}$

$\sqrt[4]{16^3} \cdot 8x^{-1/2}$

Calculator! $\frac{8}{x^{1/2}}$ Express as a radical

**Rational exponents are useful when multiplying and dividing radicals that do NOT have the same index!

Example 2: Simplify using rational exponents. Keep your answers in (simplest) radical form

a.) $\sqrt{3} \cdot \sqrt[4]{3}$

Rewrite each using Rational Exponents!

$3^{1/2} \cdot 3^{1/4}$

$3^{1/2 + 1/4} \rightarrow 3^{3/4}$

→ Rewrite using radicals!

$\sqrt[4]{3^3} \rightarrow \sqrt[4]{27}$

b.) $\frac{\sqrt[3]{36}}{\sqrt{6}}$ NEED TO CHANGE 36 into a base of 6!

$\frac{\sqrt[3]{6^2}}{\sqrt{6}}$ Now rewrite with rational exponents!

$\frac{6^{2/3}}{6^{1/2}} \rightarrow 6^{2/3 - 1/2} \rightarrow 6^{1/6}$

→ Rewrite as a radical!

$\sqrt[6]{6}$

c.) $\sqrt[4]{\frac{1}{x^3}} \cdot \sqrt[8]{x^2}$

$\sqrt[4]{x^{-3}} \cdot \sqrt[8]{x^2}$

$x^{-3/4} \cdot x^{2/8} \rightarrow x^{-3/4 + 2/8}$

$\rightarrow x^{-1/2} \rightarrow \frac{1}{x^{1/2}}$

→ Rewrite as a radical

$\frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \rightarrow \frac{\sqrt{x}}{x}$

Must rationalize

d.) $\frac{\sqrt{x^4 y}}{\sqrt[4]{x^2 y^6}}$

$\frac{x^{4/2} y^{1/2}}{x^{2/4} y^{6/4}}$

$x^{4/2 - 2/4} y^{1/2 - 6/4}$

$\rightarrow x^{3/2} y^{-1} \rightarrow \frac{x^{3/2}}{y}$

Rewrite as radical!

$\frac{\sqrt{x^3}}{y} \rightarrow \frac{x\sqrt{x}}{y}$