

**Basic Radical Property** → If have  $a^n = b$ , then "a is the nth root of b" or  $a = \sqrt[n]{b}$

where b is the called the radicand, n is the index number (degree of root), and  $\sqrt{\quad}$  is the radical symbol.

**Ex:** ▪ If  $6^2 = 36$ , then 6 is the Square Root of 36 (in words) or  $6 = \sqrt{36}$  (radical form)

▪ If  $(a^2)^3 = a^6$ , then  $a^2$  is the Cube Root of  $a^6$  (in words) or  $a^2 = \sqrt[3]{a^6}$  (radical form)

▪ If  $(b^3)^4 = b^{12}$ , then  $b^3$  is the Fourth Root of  $b^{12}$  (in words) or  $b^3 = \sqrt[4]{b^{12}}$  (radical form)

$$b^3 \cdot b^3 \cdot b^3 \cdot b^3 = b^{12}$$

• To simplify radical expressions, do the following

- 1.) Split the radicand apart (using a factor tree or the common powers chart) into factors.
- 2.) Goal of splitting the radicand apart is to get factor's power to match the index number.
- 3.) Pull out of radical → factors that match the index (exponent will be a understood 1 when pulled out)  
Remain in radical → factors that do NOT match the index (exponent will be lower than the index)

**Example 1: Simplify each radical expression.**

a.) $\sqrt{16x^2}$ $4x$	b.) $\sqrt[3]{27x^6}$ $3x^2$	c.) $\sqrt[4]{256y^{12}}$ $4y^3$	d.) $\sqrt{75a^3}$ $5a\sqrt{3a}$	e.) $\sqrt[3]{320a^2b^7}$ $4b^2\sqrt[3]{5a^2b}$	f.) $\sqrt[4]{48a^8b^{10}}$ $2a^2b^2\sqrt[4]{3b^2}$
g.) $3\sqrt{72m^5n^6}$ $18m^2n^3\sqrt{2m}$		h.) $-4\sqrt[3]{96m^4n^8}$ $-8mn^2\sqrt[3]{12mn^2}$		i.) $5\sqrt[4]{405m^6n^{11}}$ $15mn^2\sqrt[4]{5m^2n^3}$	

**Multiplying Radicals** → If have  $\sqrt[n]{a} \cdot \sqrt[n]{b}$ , then  $\sqrt[n]{ab}$  (Note: index numbers must equal!)

**Example 2: Multiply and then simplify the product.**

a.) $\sqrt{18x^2} \cdot \sqrt{2x^4}$ $\sqrt{36x^6}$ $2 \cdot 3 \cdot x^3$ $6x^3$	b.) $\sqrt[3]{4a^4} \cdot \sqrt[4]{4a^5}$ Not possible, b/c index is not the same.	c.) $4\sqrt{12x^3y} \cdot \sqrt{5xy^4}$ $4\sqrt{60x^4y^5}$ $2 \cdot 2 \cdot x^2 \cdot y^2 \sqrt{3 \cdot 5 \cdot y}$ $8x^2y^2\sqrt{15y}$	d.) $2\sqrt[3]{9a^2b^4} \cdot -5a\sqrt[3]{6a^4b}$ $-10a\sqrt[3]{54a^6b^5}$ $-10a \cdot 3 \cdot a^2b \sqrt[3]{2b^2}$ $-30a^3b\sqrt[3]{2b^2}$
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Index = 2

$$x^3 \rightarrow x \cdot x \cdot x \rightarrow x\sqrt{x} \quad \text{grouping by index}$$

$$x^3 \rightarrow x^2 \cdot x \rightarrow x\sqrt{x} \quad \text{matching the index}$$

$$x^3 \rightarrow \frac{3}{2} = 1 \text{ r } 1 \quad x\sqrt{x} \quad \text{dividing by index}$$

Only for variables

$$\sqrt[3]{x^{17}}$$

$$\underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x}_{x^3} \cdot x \cdot x \quad x^5 \sqrt[3]{x^2}$$

$$\underbrace{x^3}_{x^3} \cdot \underbrace{x^3}_{x^3} \cdot \underbrace{x^3}_{x^3} \cdot \underbrace{x^3}_{x^3} \cdot \underbrace{x^3}_{x^3} \cdot x^2 \quad x^5 \sqrt[3]{x^2}$$

$$x^{17} \rightarrow \frac{17}{3} = 5 \text{ r } 2 \quad x^5 \sqrt[3]{x^2}$$

Power  
Index

### Example #1

a) Index = 2

$$\sqrt{16x^2}$$

$x^2 = \frac{x \cdot x}{x}$

$$2 \cdot 2 \cdot x$$

$$\boxed{4x}$$

b)

$$\sqrt[3]{27x^6}$$

$x^6 = \frac{6}{3} = 2$

Exponent  
Index

$$3 \cdot x \cdot x$$

$$\boxed{3x^2}$$

c)

$$\sqrt[4]{256y^{12}}$$

$y^{12} = \frac{12}{4} = 3$

$$2 \cdot 2 \cdot y \cdot y \cdot y$$

$$\boxed{4y^3}$$

Example 1 (cont):

d)  $\sqrt{75a^3}$

$3 \uparrow 25$   $a^3 = \frac{3}{2} = 1r1$

$5 \uparrow 5$

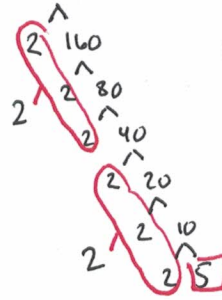
$5 \cdot a \cdot \sqrt{3 \cdot a}$

$5a\sqrt{3a}$

e)  $\sqrt[3]{320a^2b^7}$

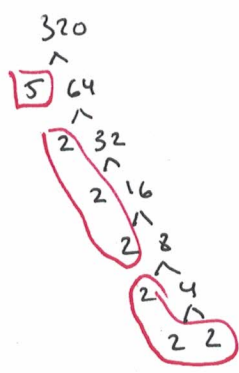
$a^2 = \frac{2}{3} \times$   $b^7 = \frac{7}{3} = 2r1$

*If exponent under  $\sqrt{\quad}$  is less than index, nothing comes out!*



$2 \cdot 2 \cdot b \cdot b \cdot \sqrt[3]{5 \cdot a^2 \cdot b}$

$4b^2\sqrt[3]{5a^2b}$



f)  $\sqrt[4]{48a^8b^{10}}$

$2 \uparrow 24$   $a^8 = \frac{8}{4} = 2$

$2 \uparrow 12$   $b^{10} = \frac{10}{4} = 2r2$

$2 \uparrow 6$   $2 \uparrow 5$

$2 \cdot a \cdot a \cdot b \cdot b \sqrt[4]{3 \cdot b \cdot b}$

$2a^2b^2\sqrt[4]{3b^2}$

g)  $\sqrt[3]{72m^5n^6}$

$2 \uparrow 36$   $m^5 = \frac{5}{2} = 2r1$

$2 \uparrow 18$   $n^6 = \frac{6}{2} = 3$

$2 \uparrow 9$   $3 \uparrow 3$

$3 \cdot 2 \cdot 3 \cdot m^2 \cdot n^3 \cdot \sqrt{2 \cdot m}$

$18m^2n^3\sqrt{2m}$

h)  $-4\sqrt[3]{96m^4n^8}$

$2 \uparrow 48$   $m^4 = \frac{4}{3} = 1r1$

$2 \uparrow 24$   $n^8 = \frac{8}{3} = 2r2$

$2 \uparrow 12$   $2 \uparrow 6$   $2 \uparrow 3$

$-4 \cdot 2 \cdot m \cdot n^2 \sqrt[3]{2 \cdot 2 \cdot 3 \cdot m \cdot n^2}$

$-8mn^2\sqrt[3]{12mn^2}$

i)  $5\sqrt[4]{405m^6n^{11}}$

$3 \uparrow 135$   $m^6 = \frac{6}{4} = 1r2$

$3 \uparrow 45$   $n^{11} = \frac{11}{4} = 2r3$

$3 \uparrow 15$   $3 \uparrow 5$

$5 \cdot 3 \cdot m \cdot n^2 \sqrt[4]{5 \cdot m^2 \cdot n^3}$

$15mn^2\sqrt[4]{5m^2n^3}$