3.1b Operations with Polynomials: Factoring Polynomials

FACTORING is the process of breaking down or breaking part a PRODUCT into its factors.

Factoring of a Polynomial is the process of breaking apart a polynomial into the factors / terms that were originally multiplied together to form the polynomial.

$$24 \;(\text{Product}) \quad \left.\begin{matrix} 1\cdot 24 \\ 2\cdot 12 \\ 3\cdot 8 \\ 4\cdot 6 \end{matrix}\right\} \text{FACTORS}$$

Polynomial → $\underbrace{x^2+5x+6}_{\text{(Product)}} \xrightarrow{\text{Factor}} \underbrace{(x+2)(x+3)}_{\text{(Factors)}}$

Polynomial → $3x^2+9xy+12x \xrightarrow{\text{Factor}} \underbrace{3x(x+3y+4)}_{\text{(Factors)}}$

There are several different **TYPES & WAYS** to factor.

❖ **THE FIRST RULE OF FACTORING IS** ALWAYS check for a GCF.

- **"GCF"**: Greatest Common Factor; when you can ONLY factor out 1-term.

  1.) Find the "largest" GCF that will go into all the coefficients (#'s attached to variables) and constant (# without a variable). Find the largest exponent all powers have in common.

****THIS IS THE GCF OF THE POLYNOMIAL****

  2.) Place the GCF in front of a set of ( ) and "factor it out of the original. (divide it out) Ask yourself the question: "What times the GCF equals the original?"

Example 1: Factor by factoring out a GCF.

a) $-6b^4 - 12b^2$	b) $-7n^6m + 2n^6 + 3n^5$	c) $72y^5z^2x^2 - 80y^2zx^3 + 16y^3z^2$
① What is the biggest # that goes into all terms? -6 ② What is the largest power that goes into all terms? b^2 GCF: $-6b^2$ ③ Now factor it out! $-6b^2(b^2+2)$	① No # other than 1. ② n^5 GCF: n^5 $n^5(-7nm+2n+3)$	① 8 is largest # ② y^2z is largest powers GCF: $8y^2z$ $8y^2z(9y^3zx^2-10x^3+2yz)$

GCF factored out!

- **Factor by Grouping**: Use this method when you have 4-term polynomials.

  1.) "Group" the 1st two terms and the last two terms (including any signs of its terms) by putting (parenthases) around each set of terms.

  2.) Factor out the GCF of each "group".

  3.) The "leftovers" in the groups should be EXACTLY the SAME.

  4.) Your final answer, **the factored polynomial**, will be the 2 GCF's times the "left over" group.

Ex. 2: Factor by Grouping

a) $2m^3 + 6m^2 + 3m + 9$	b) $27b^2 + 45b - 3b - 5$	c) $12x^3 + 10x^2 - 36x - 30$
*✓ for a GCF ① $(2m^3+6m^2)(+3m+9)$ ② $2m^2(m+3)+3(m+3)$ ③ should be the same! ④ $\boxed{(2m^2+3)(m+3)}$	*✓ for a GCF! $(27b^2+45b)(-3b-5)$ $9b(3b+5)-1(3b+5)$ $\boxed{(9b-1)(3b+5)}$	✓ for a GCF! YES! $2[6x^3+5x^2-18x-15]$ $2[(6x^3+5x^2)(-18x-15)]$ $x^2(6x+5)-3(6x+5)$ $\boxed{2(x^2-3)(6x+5)}$

- **Trinomials**: are 3-term polynomial generally in the form $ax^2 \pm bx \pm c$.

There several different ways to factor this type of polynomial: "guess and check", "Slide-Divide, Bottoms-up", "reverse box method", and "X-Box Method".

  1.) Make sure the trinomial is in Standard Form $ax^2 \pm bx \pm c$.

  2.) Before factoring this type, ALWAYS check for a GCF and factor it out first! This will be in your final answer so DO NOT FORGET IT!

- **Special Cases**: Perfect Square Trinomials and Difference of Squares

Can be factored like trinomials and there are "short cuts" for both types.

❖ Sometimes polynomials CANNOT be factored. These types of polynomials are PRIME

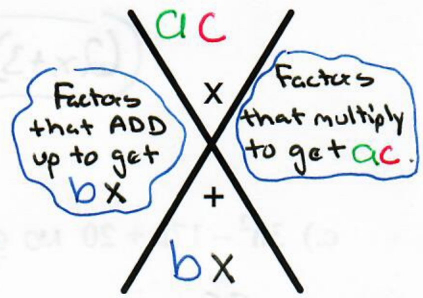
because the polynomial has only TWO factors; one and itself. ie... $x^2 + 2x + 1$, $x^2 - 3x + 13$, $x^2 + 16$ } PRIME

Factoring by "X-Box Method"

Steps to Follow when Factoring BY "REVERSE BOX METHOD"

*You should know to use this form of factoring because it works with ALL **TRINOMIALS**; except PRIME!

1) Polynomials will be in the following form: $ax^2 \pm bx \pm c$
2) **Check** for a GCF and if there is Factor it out !
3) Find two #'s that multiply to get ac and ADD up to bx.
4) Start by filling out your "box":
   - Put the ax^2 term in the upper left box and the c term in the lower right box.
   - For the rest of the "boxes" → use the "BIG X" to help you
5) Find the GCF of each row (left side) and each column (top)
6) You final answer (factored trinomial) will be the outside left column times the outside top row



GCF

GCF		
	ax^2	bx
	bx	$\pm c$

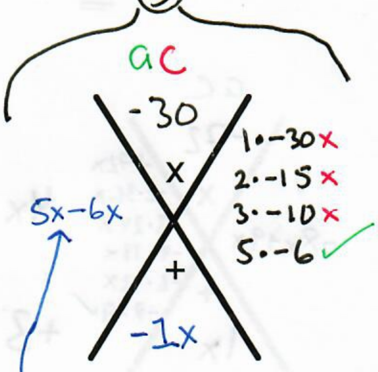
ie: $4x^2 - 2x - 30$
a b c

① ✓ for a GCF! Yes, 2!

② Factor out GCF! $2(2x^2 - x - 15)$
a b c

Don't Forget in final answer!

③



Does not matter which boxes you put these into!

(4-5)

	x	-3
$2x$	$2x^2$	$-6x$
$+5$	$5x$	-15

⑥

$2(2x+5)(x-3)$

This is factored completely!

Example 3: Factor each trinomial completely BY the "X-Box Method"

a.) $2x^2 + 11x + 12$ (a, b, c) NO GCF

ac: 24

x

$3x + 8x$

$1 \cdot 24$ ×
$2 \cdot 12$ ×
$3 \cdot 8$ ✓
$4 \cdot 6$ ×

+

$11x$

bx

	x	$+4$
$2x$	$2x^2$	$8x$
$+3$	$3x$	$+12$

$(2x+3)(x+4)$

b.) $5w^2 - 14w - 3$ ← NO GCF

ac: -15

x

$1w - 15w$

$1 \cdot -15$ ✓
$3 \cdot -5$ ×

+

$-14w$

bx

	w	-3
$5w$	$5w^2$	$-15w$
$+1$	$1w$	-3

$(5w+1)(w-3)$

c.) $3h^2 - 17h + 20$ NO GCF

ac: 60

x

$-5h - 12h$

$-1 \cdot -60$ ×
$-2 \cdot -30$ ×
$-3 \cdot -20$ ×
$-4 \cdot -15$ ×
$-5 \cdot -12$ ✓
$-6 \cdot -10$ ×

+

$-17h$

bx

	$3h$	-5
h	$3h^2$	$-5h$
-4	$-12h$	$+20$

$(h-4)(3h-5)$

d.) $4m^2 - 25$ (a, c) NO GCF and $bx = 0$

ac: -100

x

$10m - 10m$

$1 \cdot -100$ ×
$2 \cdot -50$ ×
$4 \cdot -25$ ×
$5 \cdot -20$ ×
$10 \cdot -10$ ✓

+

$0m$

bx

	$2m$	-5
$2m$	$4m^2$	$-10m$
$+5$	$10m$	-25

$(2m-5)(2m+5)$

Difference of Squares!

e.) $x^4 - 7x^2 - 18$ NO GCF

ac: -18

x

$2x^2 - 9x^2$

$1 \cdot -18$ ×
$2 \cdot -9$ ✓
$3 \cdot -6$ ×

+

$-7x^2$

	x^2	-9
x^2	$1x^4$	$-9x^2$
$+2$	$2x^2$	-18

$(x^2+2)(x^2-9)$ ← NOT COMPLETELY FACTORED!

Difference of Squares!

$(x^2+2)(x+3)(x-3)$

f.) $36x^3 + 3x^2 - 18x$ YES GCF! → $3x[12x^2 + x - 6]$ (a, b, c)

ac: -72

x

$-8x + 9x$

$-1 \cdot 72$ ×
$-2 \cdot 36$ ×
$-3 \cdot 24$ ×
$-4 \cdot 18$ ×
$-6 \cdot 12$ ×
$-8 \cdot 9$ ✓

+

$1x$

bx

	$3x$	-2
$4x$	$12x^2$	$-8x$
$+3$	$9x$	-6

$3x(4x+3)(3x-2)$