3.1 – Operations with Polynomials

- **Terms**: The number of expressions combined/separated by addition "+" or subtraction "−" symbols. Terms can be a single number "3", variables "x", and/or variables w/coefficients 2y

- **Monomial**: is a number, variable, or the variable w/coefficient.

- **Degree of a Monomial**: Is the POWER of the term. The degree is determined by the sum of ALL _____ of the exponents of the variables.

Name of Degree	Examples	Name of Degree	Examples
0 Degree Constant:	9 -3 8 * No variable	1st Degree Linear:	$3x^1$ $-19b^1$ y^1
2nd Degree Quadratic:	$-2x^2$ $5xy$ 1+1=2	3rd Degree Cubic:	x^3 $-4a^2b^1$ $23wxy$ 1+1+1=3
4th Degree Quartic:	$9x^4$ $-13xyz^2$ 1+1+2=4	5th Degree Quintic:	x^5 $32y^3z^2$ 3+2=5
"n"th Degree for degrees 6 or greater:	$10wx^2y^4z$ 7th degree	$-9a^3b^2c^5d$ 11th degree	

-Polynomial: Is a monomial or the sum and or subtraction of two or more monomials. Thus, polynomial means "many terms". Again, the "+" and "−" signs separate terms.

ie: $x^2 + 2x + 4$ ← 3 terms { $24x^2yz^3 + x^2y - y^3 + xz + 1$ ← 5 terms

Standard Form of a Polynomial: when the terms of a polynomial are written from HIGHEST degree to the lowest degree. If one or more term has the same degree, then write in "ABC" order. A polynomial has to be SIMPLIFIED completely to be in Standard Form.

Polynomials can be Classified based on the number of terms and its degree.

Number of Terms	Name Based on # of Terms	Example	Number of Terms	Name Based on # of Terms	Example
1	Monomial	2y	3	Trinomial	$4x^2 + y - 3$
2	Binomial	$-5y + 2$	4 or more	# Term Polynomial	$x^4 - 2x^3 - 5x^2 + 11x + 7$

"5 term" polynomial

Degree of a Polynomial: is = to the highest degree term. You DO NOT add up all degrees to determine the degree of the polynomial.

Leading Coefficient is the coefficient of the highest degree term of the polynomial.

Example 1: **Write each polynomial in Standard Form and classify it based on # of terms and degree.**

		Standard Form	Classification
a.)	$3a^2 + 5a - 4 + 5a^2 - 8$ * must simplify *	$8a^2 + 5a - 12$	(Degree) (# of terms) Quadratic Trinomial
b.)	$4x - (3 - 2x) + 3x$ * must simplify *	$4x - 3 + 2x + 3x \rightarrow 9x - 3$	Linear Binomial
c.)	$5w^2 - 4 + 2w^2 + 3w^6$	$3w^6 + 7w^2 - 4$	6th Degree Trinomial
d.)	$6y + 2y(4xy - 3)$	$6y + 8xy^2 - 6y \rightarrow 8xy^{1+2=3}$	Cubic Monomial

Operations with Polynomials include: Addition, Subtraction, Multiplication, and Division.

When performing operations on polynomials make sure that completely simplified and write the answers Standard Form.

- Adding & Subtracting Polynomials

1) When adding polynomials drop the ( ) and combine like terms.

2) When subtracting polynomials distribute the "−" to all terms in ( ), then drop the ( ) and combine like terms.

Example 2: **Add or subtract each polynomial and write your answer in STANDARD FORM.**

a.) $(4x^2 + 6x + 7) + (1 + 2x^2 - 9x)$

$4x^2 + 2x^2 + 6x - 9x + 7 + 1$

$6x^2 - 3x + 8$

b.) $(2a^3 - 3a + 5a^2) - (a^3 - 8a^2 + 11)$

$2a^3 - 3a + 5a^2 - a^3 + 8a^2 - 11$

$2a^3 - a^3 + 5a^2 + 8a^2 - 3a - 11$

$a^3 + 13a^2 - 3a - 11$

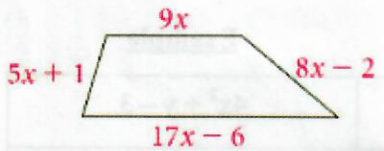
c.) $(x^2 + 2x - 4x^2) - (5x + 6x^3) + (7x^2 + 3 - 8x)$

$x^2 + 2x - 4x^2 - 5x - 6x^3 + 7x^2 + 3 - 8x$

$-6x^3 - x^2 - 4x^2 + 7x^2 + 2x - 5x - 8x + 3$

$-6x^3 + 2x^2 - 11x + 3$

d.) Find the perimeter of the figure: * Add all sides together!



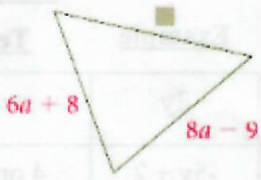
$P = (5x+1) + (9x) + (8x-2) + (17x-6)$

$5x + 9x + 8x + 17x + 1 - 2 - 6$

$39x - 7$

$P = 39x - 7$ units

d.) Find the missing length given the perimeter = $23a - 7$



$(6a+8) + (8a-9) + x = 23a - 7$

$6a + 8a + 8 - 9 + x = 23a - 7$

$14a - 1 + x = 23a - 7$

$-14a \quad +1 \qquad -14a \quad +1$

$x = 9a - 6$

The missing side is $9a - 6$ units.

- Multiplying Polynomials

1) Use the distributive property when multiplying a monomial x polynomial. $2a(3a^2+4a-5)$

2) Use either F.O.I.L or the Box Method when multiplying a binomial x binomial.

3) Use the Box Method when multiplying a binomial x trinomial.

4) Two "**SPECIAL CASES**":

a) Perfect Squares: $(x+1)^2 \rightarrow (x+1)(x+1)$ or $(x-1)^2$ $(x-1)(x-1)$

$x^2+1x+1x+1$ → x^2+2x+1

$x^2-1x-1x+1$ → x^2-2x+1

b) Difference of Squares: $(x+1)(x-1)$ * middle term cancels out, left with a "Binomial"

$x^2-1x+1x-1$

x^2-1

Example 3: **Multiply and write your answer in STANDARD FORM.**

a.) $-3y^2(3y^4-5+4y^2-2y^3)$

$-9y^6+15y^2-12y^4+6y^5$

$-9y^6+6y^5-12y^4+15y^2$

b.) $4a^2b^2(2a^3b-3ab^2+5a^5b^2)$

$8a^5b^3-12a^3b^4+20a^7b^4$

$20a^7b^4+8a^5b^3-12a^3b^4$

c.) $(7x-2)(9x-2)$

F L I O

$63x^2-14x-18x+4$

$63x^2-32x+4$

d.) $(4x-5)^2$

$(4x-5)(4x-5)$

	4x	-5
4x	$16x^2$	-20x
-5	-20x	+25

$16x^2-40x+25$

e.) $(2h+3)(2h-3)$

	2h	+3
2h	$4h^2$	6h
-3	-6h	-9

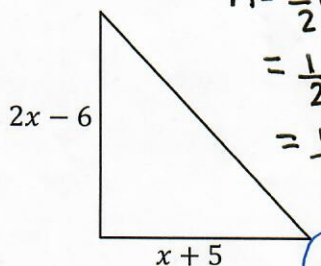
$4h^2-9$

f.) $(3x+4)(4x^2+5x-6)$

	$4x^2$	+5x	-6
3x	$12x^3$	$15x^2$	-18x
+4	$16x^2$	20x	-24

$12x^3+31x^2+2x-24$

g.) What is the area of the triangle? * $A=\frac{1}{2}bh$



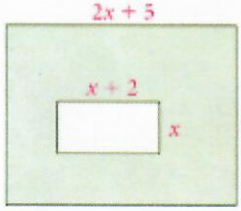
$A=\frac{1}{2}(2x-6)(x+5)$

$=\frac{1}{2}(2x^2+10x-6x-30)$

$=\frac{1}{2}(2x^2+4x-30)$

$A=x^2+2x-15$ units^2

h.) What is the area of the shaded region?



③ $\text{Area}_{\text{shade}} = \text{Area}_{\text{Big}\square} - \text{Area}_{\text{small}\square}$

$=(6x^2+10x+5)-(x^2+2x)$

$=6x^2+10x+5-x^2-2x$

$5x^2+8x+5$ units^2

① Area Big: $(2x+5)(3x+1)$

$6x^2+2x+8x+5$

$6x^2+10x+5$

② Area Small: $x(x+2)$

x^2+2x