2.6 Applications of Exponential and Logarithmic Functions

Steps to Solving Applications of Exponential & Logarithmic Equations:

1. READ the entire problem completely first.
2. Write an ANSWER SENTENCE so that you know what you are looking for. **DON'T FORGET TO INCLUDE ANY** Units of Measure. ie: $, years, % ...
3. Correctly identify which formula to use by the "Keywords" used in the problem.
4. Write the formula down, then write down all the Variables in the formula and correctly identity each value for each variable and label them. ie $A = Pe^{rt}$ Not a variable!

   A: ?
   P: 1200
   r: 2.5% → .025
   t: 6 yrs
5. Rewrite the formula by Substituting in the information found step 3.
6. Solve the equation for the missing value.
7. Before writing your answer down, does it make Sense?

Example 1: Use the appropriate formula to answer each real-world application of exponential equations.

a.) John invested $500 into an account with a 3% interest rate that is compounded quarterly. How long will it take for his investment to double?	b.) Carol invested $800 into an account that is compounded continuously. Her investment after 5 years accrued to $1,107. What was Carol's interest rate on her investment?
P, r, Key words "n", A = 1000 step 3: $A = P(1+\frac{r}{n})^{nt}$ Step 4: A: 1000 P: 500 r: 3% → .03 n: compound quarterly "4" t: ? step 5 → $1000 = 500(1+\frac{.03}{4})^{4t}$ Step 6: $\frac{1000}{500} = \frac{500(1.0075)^{4t}}{500}$ $2 = 1.0075^{4t}$ $\log 2 = \log 1.0075^{4t}$ $\frac{\log 2}{\log 1.0075} = \frac{(4t)\log 1.0075}{\log 1.0075}$ $\frac{4t}{4} = \frac{92.7657660}{4}$ $t = 23.19144152$	P, t, A, Key words $A = Pe^{rt}$ A = 1107 P = 800 r = ? t = 5 "e" is not a variable! $1107 = 800e^{r(5)}$ $\frac{1107}{800} = \frac{800e^{5r}}{800}$ $1.38375 = e^{5r}$ $\ln 1.38375 = \ln e^{5r}$ Inverses! $\frac{\ln 1.38375}{5} = \frac{5r}{5}$ $r = .064959441$ * Convert to a % for rate!

step 2: It will take 23.2 yrs for the investment to double.

$r = 6.5\%$

Carol's interest rate on her investment would be 6.5%.

c.) You bought a car for $24,000. The car's value will depreciate by 8.7% each year. In how many years will the car's value be worth about $8,800?

P, r, A

$A = P(1-r)^t$

$A = 8{,}800$
$P = 24{,}000$
$r = 8.7\% \rightarrow .087$
$t = ?$

$8{,}800 = 24{,}000(1-.087)^t$

$\frac{8{,}800}{24{,}000} = \frac{24{,}000(0.913)^t}{24{,}000}$

$11/30 = 0.913^t$

$\log(11/30) = \log(0.913^t)$

$\frac{\log(11/30)}{\log 0.913} = \frac{(t)\log(0.913)}{\log 0.913}$

$t = 11.02294815$

It will take about 11 years to depreciate to $8,800.

d.) In 1910, the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, in what year will it be approximately 564,480 people?

P, r, Key words, A

$A = P(1+r)^t$

$A = 564{,}480$
$P = 120{,}000$
$r = 1.5\% \rightarrow .015$
$t = ?$

(A) $564{,}480 = 120{,}000(1+.015)^t$

$\frac{564{,}480}{120{,}000} = \frac{120{,}000(1.015)^t}{120{,}000}$

$4.704 = 1.015^t$

$\log 4.704 \cdot \log 1.015^t$

$\frac{\log 4.704}{\log 1.015} = \frac{(t)\log 1.015}{\log 1.015}$

$t = 103.9998328$

$t = 104$

(B) Find year

1910
$+ 104$
2014

The year will be 2014.

e.) An island initially had 500 rabbits and is growing each year. If the growth rate is figured out to be 32.5% per year, after how many years will the population be about 45,000 rabbits?

P, r, A, Key words!

$A = P(1+r)^t$

$A = 45{,}000$
$P = 500$
$r = 32.5\% \rightarrow .325$
$t = ?$

$45{,}000 = 500(1+.325)^t$

$\frac{45{,}000}{500} = \frac{500(1.325)^t}{500}$

$90 = 1.325^t$

$\log 90 = \log 1.325^t$

$\frac{\log 90}{\log 1.325} = \frac{(t)\log 1.325}{\log 1.325}$

$t = 15.99008686$

$t = 16$

It will take about 16 years for the population to become 45,000 rabbits.

f.) Amber has a savings account in which her money is being compounded continuously with a 3% interest rate. After 8 years, Amber's account has a balance of $1,907. What was Amber's initial deposit for the account?

r, Key words, t, A

$A = Pe^{rt}$

$A: 1{,}907$
$P: ?$
$r: 3\% \rightarrow .03$
$t: 8$

$1{,}907 = Pe^{.03(8)}$

$\frac{1907}{e^{.24}} = \frac{Pe^{.24}}{e^{.24}}$

$P = 1500.099331$

$P = 1500.10$

Amber's initial deposit was $1500.10.

g) Desmond is investing $800 into an account with a 5% interest rate. How long will it take for the account to be $2800 if the money is compounded quarterly?

Key words

$A = P(1+\frac{r}{n})^{nt}$

$A: 2800$
$P: 800$
$r: 5\% \rightarrow .05$
$n: 4$ "quarterly"
$t: ?$

$2800 = 800(1+\frac{.05}{4})^{4t}$

$\frac{2800}{800} = \frac{800(1.0125)^{4t}}{800}$

$3.5 = 1.0125^{4t}$

$\log 3.5 = \log 1.0125^{4t}$

$\frac{\log 3.5}{\log 1.0125} = \frac{(4t)\log 1.0125}{\log 1.0125}$

$\frac{4t}{4} = \frac{100.8461221}{4}$

$t = 25.21153052$

It will take about 25.2 years.

Common Exponential Application Formulas

Compound Interest w/"n" values

$$A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$$

A = final amount

P = principle amount aka "initial" - starting - deposit - investment

r = interest rate * Comes as a % change to decimal.

n = # of times $ is compounded

t = time (always in years)

Key word(s):

"Annually"	n = 1	"Monthly"	n = 12
"Semi-anually"	n = 2	"Weekly"	n = 52
"Quarterly"	n = 4	"Daily"	n = 365

Compounded Continuously

$$A = Pe^{r\cdot t}$$

↑ NOT a VARIABLE

$e \approx 2.71828$

A = final amount

P = principle amount aka "initial" - deposit - investment

r = interest rate * change to a decimal!

e = natural base (exponential Function)

t = time

Key word(s):

Compounded "continously"

Exponential Growth

$y=a(b)^x$

$b>1$

$$A = a(1+r)^t$$

A = final amount

a = initial amount

> DOES NOT HAVE TO BE $

r = rate of growth ← % needs to convert to decimal!

t = time

Key word(s):

Increases

grows

growth rate

Appreciates

Exponential Decay

$y=a(b)^x$

$0<b<1$

$$A = a(1-r)^t$$

A = final amount

a = initial amount

> Does not have to be $

r = rate of decay ← % needs to convert to decimal

t = time

Key word(s):

Decreases

decays

decay rate

Depreciates

*Converting a % to a decimal: 2.5% ⟶ move decimal two places left!

2.5% → .025