2.5 – Exponential and Logarithmic Equations

Solving Exponential Equations with unlike bases: (Type 2 of Exponential Equations)

If you CANNOT make both sides of the equation have the same base, you then will have to take the log (common log) or the ln (natural log) of both sides. * How will I know when to use ln? When you see "e". *

***CANNOT TAKE THE log(0) OR THE log(-#) !!**

Ex. 1: REVIEW of Exponential Eqns: Type 1 – Both Sides have the Same Base

a.) $4^{3x} = 32^{x+1}$

$(2^2)^{3x} = (2^5)^{x+1}$

$2^{2(3x)} = 2^{5(x+1)}$

$2(3x) = 5(x+1)$

$6x = 5x + 5$

$-5x \quad -5x$

$x = 5$

$\{5\}$

b.) $3^{2x+5} = \left(\frac{1}{9}\right)^{x-1}$

$3^{2x+5} = \left(\frac{1}{3^2}\right)^{x-1}$

$3^{2x+5} = 3^{-2(x-1)}$

$2x+5 = -2(x-1)$

$2x+5 = -2x+2$

$+2x \quad +2x$

$4x+5 = 2$

$-5 \quad -5$

$4x = -3 \rightarrow x = -3/4$

$\{-3/4\}$

Ex. 2: Exponential Eqns: Type 2 – Both Sides NOT the Same Base → Take log (or ln) of both sides

a.) $3^{2x-1} = 7$ → How do you bring down the exponent? Log. Power Prop.

$\log 3^{(2x-1)} = \log 7$

$(2x-1)\cdot\log 3 = \log 7$

$\frac{}{\log 3} \quad \frac{}{\log 3}$

$2x-1 = \frac{\log 7}{\log 3}$

$+1 \quad +1$

$2x = \left(\frac{\log 7}{\log 3}\right) + 1$

$\frac{2x}{2} = \frac{2.771243749}{2}$

$x = 1.385621875$

$x \approx 1.386$

$\{1.386\}$

b.) $3 \cdot 4^x + 11 = 2$

$-11 \quad -11$

$\frac{3 \cdot 4^x}{3} = \frac{-9}{3}$

$4^x = -3$

$\log 4^x = \log(-3)$

$(x)\ \frac{\log 4}{\log 4} = \frac{\log(-3)}{\log 4}$

$x = \frac{\log(-3)}{\log 4}$ ← CANNOT take the log(-#).

$\varnothing$

c.) $\frac{2e^{5x-3}}{2} = \frac{16}{2}$

$e^{5x-3} = 8$

$\ln e^{5x-3} = \ln 8$

Inverses!

$5x-3 = \ln(8)$

$+3 \quad +3$

$\frac{5x}{5} = \frac{\ln(8)+3}{5}$

$x = \frac{\ln(8)+3}{5}$

$(\ln(8)+3)/5$ * In Calculator *

$x = 1.015888308$

$\{1.016\}$

d.) $(3e^x - 6)(e^x + 4) = 0$

* DO NOT BOX or FOIL b/c it is set = to 0. Use ZPP. * zero product property!

$3e^x - 6 = 0$

$+6 \quad +6$

$\frac{3e^x}{3} = \frac{6}{3}$

$e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

$x = 0.693147180$

$x = .693$

$e^x + 4 = 0$

$e^x = -4$

$\ln e^x = \ln(-4)$

$x = \ln(-4)$ ← cannot take the ln(-#). * EXTRANEOUS SOL

$\{0.693\}$

f.) $4e^{3-6x} - 13 = 9$

$+13 \quad +13$

$\frac{4e^{3-6x}}{4} = \frac{22}{4}$

$e^{3-6x} = 5.5$

$\ln e^{3-6x} = \ln 5.5$

$3 - 6x = \ln 5.5$

$-3 \quad -3$

$\frac{-6x}{-6} = \frac{\ln(5.5) - 3}{-6}$

$x = \frac{\ln(5.5)-3}{-6}$

$x = .215875318$

$x = .216$

$\{0.216\}$

g) $5^{4x+7} = 4^{2x}$

$\log 5^{4x+7} = \log 4^{2x}$ * use Power Property of Logs *

$(4x+7)\cdot\log 5 = (2x)\log 4 \rightarrow 2\log 4$

$\frac{(4x+7)\log 5}{\log 5} = \frac{1.204119983x}{\log 5}$

$4x+7 = 1.722706232x$

$-4x \quad -4x$

$\frac{7}{-2.277293768} = \frac{-2.277293768x}{-2.277293768}$

$x = -3.074$

$\{-3.074\}$

Solving Logarithmic Equations:

Like exponential equations, there are TWO types of logarithmic equations.

- **Type 1** – Are logarithmic equations where BOTH sides have a SINGLE log w/same base. If this is the type you are solving, then take "base" of both sides.
- **Type 2** – Are logarithmic equations where one single log on one side and a (#) CONSTANT on the other side. If this is the type you are solving, then do "hamburger helper" to change to an Exponential.

****IMPORTANT TO REMEMBER****

1.) Keep answers as **(reduced)** fractions when possible – if not, ROUND to 3 places! → thousandths

2.) Remember – You **CAN NOT** take log of a negative number or zero (b/c VA: $x = 0$)

3.) Some solutions **MAY OR MAY NOT WORK** (could also have no solutions)!

4.) When done solving a log equation – **Check** to see that it works in the **original problem**! * MUST ✓ your solution.

Ex. 3: Logarithmic Eqns: Type 1 – Both sides (and every term) is a logarithm

a.) $\log_5(6-3x) = \log_5(10-5x)$	b.) $\log(x-3)+\log x = \log 28$
c.) $\ln(2x+1)-\ln(x-1) = \ln 7$	d.) $\log_2(3x^2-3x) = \log_2(70+2x^2)$

Ex. 4: Logarithmic Eqns: Type 2 – SINGLE LOG on one side and a CONSTANT on the other side

a.) $\log_3(2x+15) = 2$	b.) $\log_8(4x^2-6)+\log_8 4 = 1$	c.) $4-6\ln(3x-1) = -20$
d.) $\log(6x+2)^2 = 4$	e.) $\log_6(x-4)-\log_6(x+3) = 2$	f) $\ln(2x+3)+\ln 7 = \ln e^5$