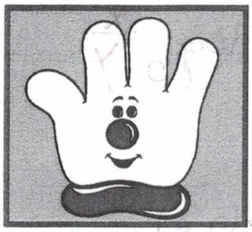
2.4 Properties of Logarithms

Properties of Logarithmic Functions

Basic Log Property: (Hamburger Helper Hand) → helps to CONVERT from LOG FORM to EXP FORM



Logarithmic Form		Exponential Form
$\log_b y = x$ (base, raised)	⟷	$b^x = y$
natural "log" → $\ln y = x$ (needs base "e")	⟷	$e^x = y$

* base e ≈ 2.718

* base "e" is not a variable!

Example 1: Rewrite each expression as either exponential or logarithmic.

Log → Exp

a.) $\log_{11} 121 = 2$ base $11^2 = 121$	b.) $\log_3 x = 4$ $3^4 = x$	c.) $\ln 2.71828 = 1$ natural "log" needs base e! $e^1 = 2.71828$
d.) $9^{-2} = \frac{1}{81}$ base of my log $\log_9 \frac{1}{81} = -2$	e.) $5^2 = 25$ $\log_5 25 = 2$	f.) $e^2 = 7.389$ $\ln 7.389 = 2$ natural base needs natural log (ln)

Exp → Log

* To check, just rewrite back to Exp. *

Special Inverses: → these pairs are INVERSES of each other and can be Simplified.

Base 10 and Common Log

10^x and $\log x$ SAME $\log_{10} x$

ie: $10^{\log 2x+3} \rightarrow 2x+3$ Inverses! $\log 10^{2x+3} \rightarrow 2x+3$

Natural Base and Natural Log ← ln

e^x and $\ln x$

$e^{\ln 5} \rightarrow 5$ $\ln e^{x+1} \rightarrow x+1$

The Ti-84 calculator can only do base 10 logarithms (aka common logs).

What if? $\log_3 9^{n+1}$ ↓ $\log_3 (3^2)^{n+1}$ ↘ $2(n+1)$

Change of Base Formula: → If you have $\log_b x$, then could use

① $\log_b x \rightarrow \frac{\log x}{\log b}$ (common log) ② $\log_b x \rightarrow \frac{\ln x}{\ln b}$ (natural log)

Example 2: Evaluate using the Change of Base Formula; round to the thousandths place if necessary.

a.) $\log_5 2$ *Use Change of Base* $\frac{\log 2}{\log 5} \rightarrow .431$	b.) $\log_6 18$ $\frac{\log 18}{\log 6} \rightarrow 1.613$	c.) $\log_2 8$ ← Can this be made into the base? $\log_2 8$ $\log_2 2^3$ 3 $\frac{\log 8}{\log 2} \rightarrow 3$

**Remember Logarithms are Exponents in disguise. ** $\log_5 125 = 3 \leftrightarrow 5^3 = 125$

Laws of Logarithms → Law # 1 (Product Rule): $\log_b x + \log_b y \longleftrightarrow \log_b (xy)$

Law # 2 (Power to Power): $\log_b x^y \longleftrightarrow y \cdot \log_b x$

Law # 3 (Quotient Rule): $\log_b x - \log_b y \longleftrightarrow \log_b \frac{x}{y}$

Example 3: Use the Laws of Logarithms to expand or condense each logarithm.

a.) $\log_2(a \cdot b^5)$ Power Expand Product $\log_2 a + \log_2 b^5 \longrightarrow \log_2 a + 5\log_2 b$	b.) $\ln\left(\frac{\sqrt{m}}{n^4 p}\right)$ Expand ← Quotient Product and Power ① $\ln\sqrt{m} - \ln(n^4 p)$ ② $\ln\sqrt{m} - (\ln(n^4) + \ln p)$ ③ $\ln\sqrt{m} - (4\ln n + \ln p)$
c.) $2\log x + \log y$ condense Power Product $\log x^2 + \log y \rightarrow \log(x^2 y)$	d.) $\log_5 a - (4\log_5 b + 2\log_5 c)$ condense quotient product $\log_5 a - (\log_5 b^4 + \log_5 c^2)$ $\log_5 a - \log_5 (b^4 c^2) \longrightarrow \log_5\left(\frac{a}{b^4 c^2}\right)$

Example 4: Evaluate each expression or find the value of x.

a.) $\log_3 9 = x$ Rewrite in Expo form. $3^x = 9$ $3^x = 3^2$ $x = 2$ $\{2\}$	b.) $\log_4 8$ ← rewrite $\log_4 8 = x$ $4^x = 8$ ← Change to base of 2 $(2^2)^x = 2^3 \rightarrow 2x = 3 \rightarrow x = 3/2$ $3/2$	c.) $\log_2\left(\frac{1}{16}\right) = x$ rewrite into Expo form $2^x = \frac{1}{16}$ $2^x = 16^{-1}$ $2^x = (2^4)^{-1}$ $\cancel{2}^x = \cancel{2}^{-4}$ $x = -4$ -4
d.) $\log_8 \sqrt{\frac{1}{2}} = x$ $8^x = \sqrt{1/2}$ $(2^3)^x = \sqrt{2^{-1}}$ $2^{3x} = (2^{-1})^{1/2}$ $3x = -\frac{1}{2}$ $x = -1/6$ rewrite as fractional exponent	e.) $\log_x 5 = \frac{1}{3}$ $x^{1/3} = 5$ $\sqrt[3]{x} = 5$ $(\sqrt[3]{x})^3 = (5)^3$ $x = 125$ 125	f.) $\log_2 112 - \log_2 7$ Quotient $\log_2 \frac{112}{7}$ $\log_2 16$ $\cancel{\log_2} 2^4 \longrightarrow 4$
g.) $\log_{12} 9 + \log_{12} 16$ Product $\log_{12}(9 \cdot 16)$ $\log_{12} 144$ $\cancel{\log_{12} 12}^2 \rightarrow 2$	h.) $e^{3\ln 2 - \ln 4}$ Power Quotient $e^{\ln 2^3 - \ln 4}$ $e^{\ln 8 - \ln 4}$ $e^{\ln \frac{8}{4}} \rightarrow \cancel{e}^{\ln 2} \rightarrow 2$	i.) $\log 4 + \log 25 - \log 1000$ $\log(4 \cdot 25) - \log 1000$ $\log 100 - \log 1000$ $\log \frac{100}{1000}$ -1

$\log \frac{1}{10} = x$ → $10^x = \frac{1}{10}$, $10^x = 10^{-1}$, $x = -1$

$\sqrt{x} \rightarrow x^{1/2}$ $\sqrt[3]{x} \rightarrow x^{1/3}$