2.5 – Binomial Expansions (Using Combinations)

- **binomial** → a two term expression which include one or more variables $2x+5$, $3x-5y$
- **binomial expansion** → a binomial raised to nth power such as $(a + b)^n$, where n is a whole number, where the coefficients are symmetric, the exponents on "a" decrease and the exponents on "b" increase, and the sum of the exponents in each term add up to "n"

$(a+b)^2 \to (a+b)(a+b)$
$a^2+ab+ab+b^2$
$a^2+2ab+b^2$

- There are three ways to show a binomial expansion:
  1.) Box Method → Organize way to expand a binomial but can be time consuming.
  2.) Pascal's Triangle → Systemic way to expand a binomial and contains multiple patterns
  3.) Combinations → Condensed way to expand a binomial and contains easier patterns
- Let's look at a simple example: $(x + 4)^3$ → Produce the answer through the 3 different methods:

Box Method	Pascal's Triangle	Combinations $_nC_r$
$(x+4)^3 \to (x+4)(x+4)(x+4)$ $\begin{array}{c\|c\|c} & x & +4 \\ \hline x & x^2 & 4x \\ \hline +4 & 4x & 16 \end{array}$ $(x+4)(x^2+8x+16)$ $\begin{array}{c\|c\|c\|c} & x^2 & +8x & +16 \\ \hline x & x^3 & 8x^2 & 16x \\ \hline +4 & 4x^2 & 32x & 64 \end{array}$ $\boxed{x^3+12x^2+48x+64}$	$(x+4)^3$ 1 1 1 1 2 1 1 3 3 1 ← $(x+4)^3$ $\begin{array}{c\|c\|c\|c\|l} 1 & 3 & 3 & 1 & \text{: coeff.} \\ (x)^3 & (x)^2 & (x)^1 & (x)^0 & \text{: 1st term} \\ (4)^0 & (4)^1 & (4)^2 & (4)^3 & \text{: 2nd term} \\ \hline x^3 & 12x^2 & 48x & 64 & \end{array}$ "multiply columns" $\boxed{x^3+12x^2+48x+64}$	$(x+4)^3$ $_3C_0\,(x)^3(4)^0 = 1\cdot x^3\cdot 1 = x^3$ $_3C_1\,(x)^2(4)^1 = 3\cdot x^2\cdot 4 = 12x^2$ $_3C_2\,(x)^1(4)^2 = 3\cdot x\cdot 16 = 48x$ $_3C_3\,(x)^0(4)^3 = 1\cdot 1\cdot 64 = 64$ $\boxed{x^3+12x^2+48x+64}$

Examples: **Complete each problem below by expanding the binomial through combinations.**

1.) Expand: $(2x + 5)^4$	2.) Find 3rd term: $(x^3 – 2)^7$	3.) Find middle term: $(3x^2 – 4y^4)^{10}$
$_4C_0\,(2x)^4(5)^0 : 1\cdot 16x^4\cdot 1 = 16x^4$ $_4C_1\,(2x)^3(5)^1 : 4\cdot 8x^3\cdot 5 = 160x^3$ $_4C_2\,(2x)^2(5)^2 : 6\cdot 4x^2\cdot 25 = 600x^2$ $_4C_3\,(2x)^1(5)^3 : 4\cdot 2x\cdot 125 = 1000x$ $_4C_4\,(2x)^0(5)^4 : 1\cdot 1\cdot 625 = 625$ $\boxed{16x^4+160x^3+600x^2+1000x+625}$	$_7C_0\,(x^3)^7(-2)^0$ $_7C_1\,(x^3)^6(-2)^1$ $\boxed{_7C_2\,(x^3)^5(-2)^2}$ ↓ $21\cdot x^{15}\cdot 4$ $\boxed{84x^{15} \text{ is 3rd term}}$	$\frac{10}{2} = 5$ $_{10}C_5\,(3x^2)^5(-4y^4)^5$ ↓ $252\cdot 243x^{10}\cdot -1024y^{20}$ $\boxed{-62{,}705{,}664x^{10}y^{20}}$