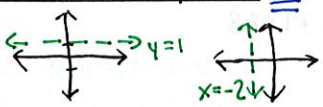
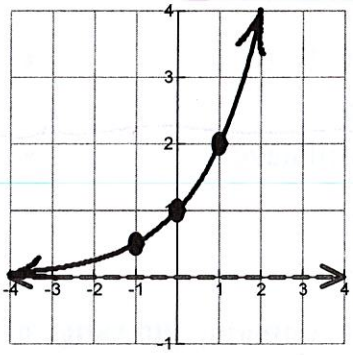
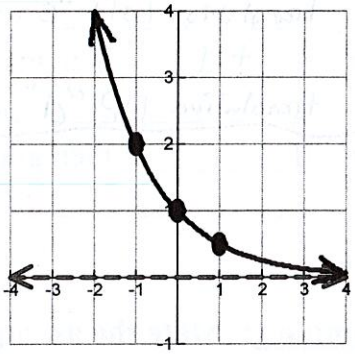
2.3 – Exponential and Logarithmic Functions

An Asymptote is a vertical ↕ x = # or horizontal ↔ y = # line that a function's graph will NOT cross. * An asymptote is represented by a dotted line on a graph.



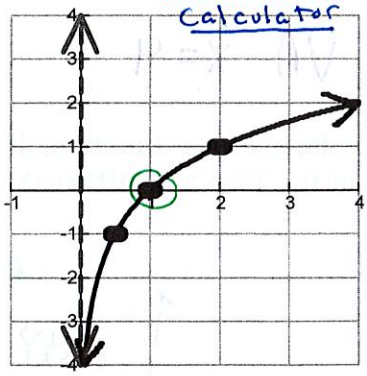
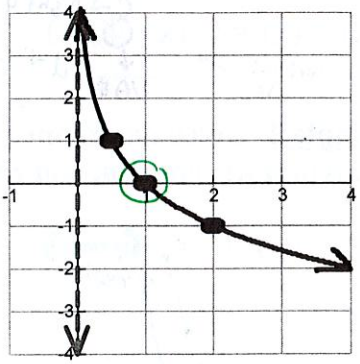
Exponential / Logarithmic Functions and Their Characteristics

➢ **Exponential function** → a function in the form $y = (b)^x$ where $b > 0$, $b \neq 1$, and x is IR. (All real #'s)

Exponential Function's Characteristics		Graphs of Exponential Functions	
Domain: $(-\infty, \infty)$	Range: $(0, \infty)$ AKA (HA, ∞)	a.) Graph of $y = (2)^x$	b.) Graph of $y = (\frac{1}{2})^x$
Common Pt: $(0, 1)$	Horizontal Asymptote: HA $y = 0$		
Note: If graph has a vertical transformation $(\pm d)$ or Reflection (-a) over x-axis, then the RANGE **AND** HA # are effected.			

Inverses of each other

➢ **Logarithmic function** → a function in the form $y = \log_b(x)$ where $b > 0$, $b \neq 1$, and x is > 0.

Logarithmic Function's Characteristics		Graphs of Exponential Functions	
Domain: $(0, \infty)$ AKA (VA, ∞)	Range: $(-\infty, \infty)$	a.) Graph of $y = \log_2(x)$ cannot put in calculator	b.) Graph of $y = \log_{\frac{1}{2}}(x)$
Common Pt: $(1, 0)$	Vertical Asymptote: VA $x = 0$		
Note: If graph has a horizontal transformation or Reflection over y-axis, then the Domain **AND** VA # are effected.			

❖ If base > 1, the graph is increasing or "growing".

❖ If 0 < base < 1, the graph is decreasing or "decaying".

❖ A MAJOR characteristic between exponential and logarithmic functions is that they are INVERSES of each other.

$\log x \longleftrightarrow \log_{10} x$

- 10^x (base 10) and $\log x$ (common log) are inverses of each other
- e^x (natural base) and $\ln x$ (natural log) are inverses of each other

$e^1 \approx 2.71828$

Transforming Exponential & Logarithmic Graphs

$y = a \cdot (base)^{bx \pm c} \pm d$ or $y = a \cdot \log_{base}(bx \pm c) \pm d$

- If $a < 0$ then graph will have a reflection over the x-axis.
- If $a > 1$ then graph will have a vertical stretch by "a".
- If $+c$ then graph will have a translation Left "c" units.
- If $+d$ then graph will have a translation up "d" units.
- If _____ then graph will have a _____.

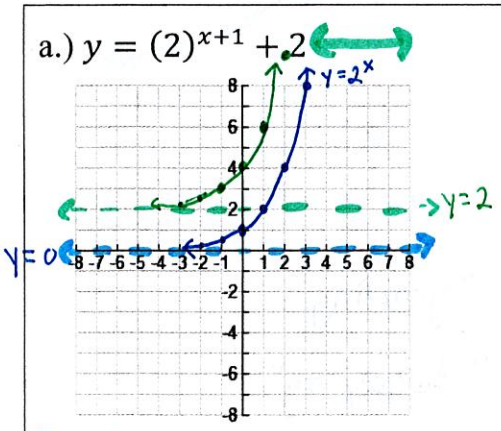
- If ////// then graph will have a //////.
- If $0 < a < 1$ then graph will have a vertical compression by "a".
- If $-c$ then graph will have a translation Right "c" units
- If $-d$ then graph will have a translation down "d" units.
- If _____ then graph will have a _____.

Example 1: State the asymptote, domain, and range of each given function using interval notation.

	Given Exp / Log Function	Asymptote	Domain	Range
Exp	a.) $f(x) = 4^{x-3} + 5$ (base; c→right 3 units; +5→HA #; d→up 5 units)	HA $y = 5$	$(-\infty, \infty)$	$(5, \infty)$ (HA, ∞)
Log	b.) $f(x) = \log_3(x + 4) - 3$ (base; c→left 4; VA; d→down 3)	VA $x = -4$	$(-4, \infty)$ (VA, ∞)	$(-\infty, \infty)$
Exp	c.) $f(x) = (\frac{1}{3})^{x+5} - 2$ (base; c→left 5; −2→HA #; d→down 2 units)	HA $y = -2$	$(-\infty, \infty)$	$(-2, \infty)$
Natural Log	d.) $f(x) = \ln(x - 4) + 1$ (natural log; c→right 4; VA #; d→up 1)	VA $x = 4$	$(4, \infty)$	$(-\infty, \infty)$

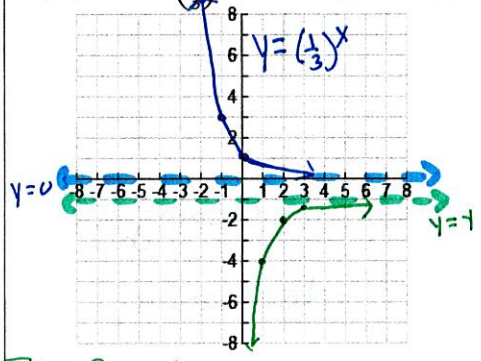
Example 2: Given an exponential or logarithmic function, draw the parent AND transformed graph. State the asymptote, domain, and range of the TRANSFORMED graph on provided lines.

a.) $y = (2)^{x+1} + 2$



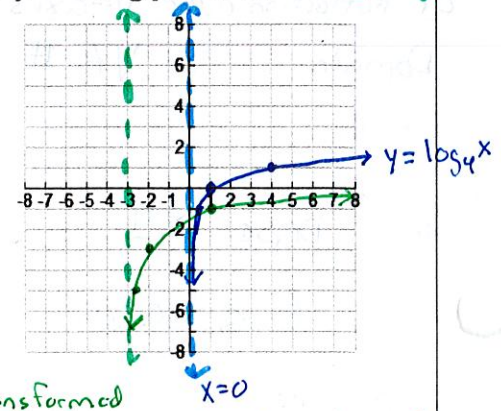
Transformed
HA $y = 2$ D: $(-\infty, \infty)$ R: $(2, \infty)$

b.) $y = -\left(\frac{1}{3}\right)^{x-2} - 1$



Transformed
HA $y = -1$ D: $(-\infty, \infty)$ R: $(-\infty, -1)$

c.) $y = 2\log_4(x + 3) - 3$



Transformed
VA $x = -3$ D: $(-3, \infty)$ R: $(-\infty, \infty)$

* SEE SEPARATE SHEET FOR WORK *

2.3 Notes (continued)

Example 2

Expo a) $y = (2)^{x+1} + 2$ ("a", "c", "d"; HA #) → Parent function

Transformed function

$y = (2)^x$ Asymptote: HA $y = 0$

x	y
-1	½
0	1
1	2
2	4
3	8

Asymptote HA $y = 2$

Domain: $(-\infty, \infty)$

Range: $(2, \infty)$ $(HA\#, \infty)$

Transformations

"c" left 1 unit

"d" up 2 units

Expo b) $y = -(1/3)^{x-2} - 1$ ("a", c, d; HA #) → Parent function

Transformed Function

$y = (1/3)^x$ Asymptote: HA $y = 0$

x	y
-2	9
-1	3
0	1
1	1/3

Asymptote HA $y = -1$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -1)$

Changes direction & range! *

Transformations

"a" < 0 reflect x-axis

"c" right 2 units

"d" down 1 unit

Log

"a" "c" VA "d"

c) $y = 2\log_4(x+3) - 3$ → parent function $y = \log_4(x)$ ⟷ Inverse $y = 4^x$

Transformed Function

can not put into calculator!

x	y
1/4	-1
1	0
4	1
16	2

x	y
-1	1/4
0	1
1	4
2	16

Asymptote: VA $x = -3$

(VA#, ∞) Domain: $(-3, \infty)$

Range: $(-\infty, \infty)$

Asymptote VA $x = 0$

x	y
1/4	-1(2) → -2
1	0(2) → 0
4	1(2) → 2
16	2(2) → 4

"a" vertical stretch by 2 *multiply "y"s by 2!

"c" left 3 units

"d" down 3 units