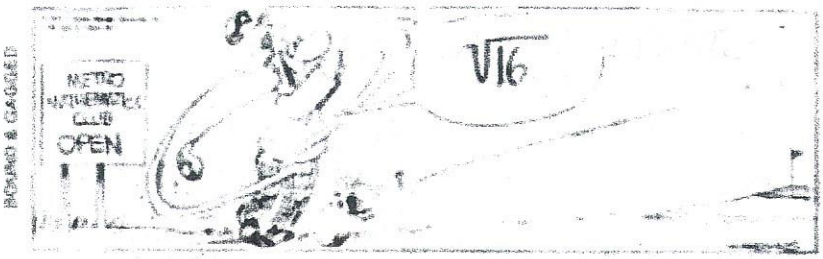
Square root (of a number) → If have $a^2 = b$, then $a = \sqrt{b}$ or $a = -\sqrt{b}$ because $(a)^2 = b$ or $(-a)^2 = b$

$\sqrt{\ }$ ↕ 2 $\sqrt{\ }$



ie $\sqrt{16} = \pm 4$

$(4)^2 = 16$ or $(-4)^2 = 16$

$\sqrt{\ }$ ← Radical

- **radicand** – represents the expression or # under the $\sqrt{\ }$ Ex: $\sqrt{4}$ where 4 is the radicand
- **index number** – represents the small # located inside $\sqrt{\ }$ Ex: $\sqrt[3]{x}$ where index = 3 "cubed root"
- Notes: 1.) You can have a POSITIVE SQUARE ROOT (called the PRINCIPAL sq. root $\sqrt{\ } = +\sqrt{\ }$)
  - 2.) You can have a NEGATIVE Sq. ROOT where negative sign is in front of $\sqrt{\ } = -\sqrt{\ }$
  - 3.) (Normally) You have a positive and negative sq. root where a $\pm$ symbol is used $\pm\sqrt{\ }$
  - 4.) (Occasionally) You can have a negative radicand where answer is Imaginary Solution (NO REAL SOLUTION)
- GOAL to simplifying square roots – take out Perfect Square Factors where the easiest way to do this is by breaking apart the radicand using a Factor Tree; "Birthday Cake Method"

Example 1: Simplify each radical (square root) completely. NO DECIMALS, EXACT ANSWERS ONLY!

2 is index; square root

a.) Simplify: $\sqrt{18}$

18 → 2, 9 → 3, 3

$3 \cdot \sqrt{2}$

$3\sqrt{2}$

b.) Simplify: $\sqrt{27}$

27 → 3, 9 → 3, 3

$3 \cdot \sqrt{3}$

$3\sqrt{3}$

c.) Simplify: $\sqrt{360}$

360 → 2, 180 → 2, 90 → 2, 45 → 3, 15 → 3, 5

$2 \cdot 3 \cdot \sqrt{2 \cdot 5}$

$6\sqrt{10}$

d.) Simplify: $5\sqrt{48}$

48 → 2, 24 → 2, 12 → 2, 6 → 2, 3

$5 \cdot 2 \cdot 2 \cdot \sqrt{3}$

$20\sqrt{3}$

e.) Simplify: $-3\sqrt{32}$

32 → 2, 16 → 2, 8 → 2, 4 → 2, 2

$-3 \cdot 2 \cdot 2 \cdot \sqrt{2}$

$-12\sqrt{2}$

f.) Simplify: $3\sqrt{5} \cdot 2\sqrt{10} = 6\sqrt{50}$

50 → 2, 25 → 5, 5, 5

$6 \cdot 5 \cdot \sqrt{2}$

$30\sqrt{2}$

$\sqrt{2 \cdot 25} \rightarrow 5\sqrt{2} \cdot 6 = 30\sqrt{2}$

Follow these steps to solve: Step 1 – Isolate the x^2 term

Step 2 – Take the $\sqrt{\ }$ of x^2 and $\pm\sqrt{\ }$ of the term on the right side of the $=$.

Step 3 – SIMPLIFY the radicand

$\sqrt{8} \rightarrow 2\sqrt{2}$

* Notes: Keep answers in simplified radical form

If get a negative underneath square root → answer is ~~no real solution~~ Complex Ex: $x = \sqrt{-2}$ $x = \pm i\sqrt{2}$

Example 2: Solve each quadratic equation by using square roots. EXACT SOLUTIONS! $\sqrt{-1} = i$

a.) $3x^2 = 75$

$\frac{3}{3} \quad \frac{}{3}$

$x^2 = 25$

$\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

$\{\pm 5\}$

b.) $24x^2 - 6 = 0$

$+6 \quad +6$

$\frac{24x^2}{24} = \frac{6}{24}$

$x^2 = \frac{1}{4}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{4}}$

$x = \pm \frac{1}{2}$

$\{\pm \frac{1}{2}\}$

c.) $6 - 4x^2 = -18$

$-6 \quad -6$

$\frac{-4x^2}{-4} = \frac{-24}{-4}$

$x^2 = 6$

$\sqrt{x^2} = \pm\sqrt{6}$

$x = \pm\sqrt{6}$

$\{\pm\sqrt{6}\}$

d.) $8x^2 - 10 = 214$

$+10 \quad +10$

$\frac{8x^2}{8} = \frac{224}{8}$

$x^2 = 28$

$\sqrt{x^2} = \pm\sqrt{28}$

$x = \pm 2\sqrt{7}$ ($28 = 4 \cdot 7$, $\sqrt{4} = 2$)

$\{\pm 2\sqrt{7}\}$

e.) $2x^2 + 16 = 0$

$-16 \quad -16$

$\frac{2x^2}{2} = \frac{-16}{2}$

$x^2 = -8$

$\sqrt{x^2} = \pm\sqrt{-8}$

$\sqrt{-1} \cdot \sqrt{8}$

$i \cdot 2\sqrt{2}$ ($8 = 4 \cdot 2$, $\sqrt{4} = 2$)

$x = \pm 2i\sqrt{2}$

$\{\pm 2i\sqrt{2}\}$

Two solutions; a quadratic has up to two solutions.

- **quadratic equation # 2 (to solve by sq roots using a "formula")** → equation needs to be in the form:

$ax^2 \pm bx \pm c = 0$ where $x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$ (this is called the quadratic formula)

* Notes: This method will work for ANY QUADRATIC EQUATION but the equation must $= 0$ Standard Form

Keep answers in simplified radical form

If get a negative underneath square root → answer is ~~no real solution~~ Ex: $x = \sqrt{-2} \rightarrow x = \pm i\sqrt{2}$

Example 3: Solve each quadratic equation using the Quadratic Formula. Round to tenth place.

a.) $x^2 - 5x + 6 = 0$

$\{2, 3\}$

b.) $-2x^2 + 8 - 4x = 0$

$\{-1 \pm \sqrt{5}\}$

c.) $3x^2 = -1$

$\{\pm \frac{i\sqrt{3}}{3}\}$

d.) $2x^2 + 4x - 5 = 2$

$\{\frac{-2 \pm 3\sqrt{2}}{2}\}$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

Example 3

a) $x^2 - 5x + 6 = 0$

$a = 1 \quad b = -5 \quad c = 6$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

*Simplify one piece at a time

$$x = \frac{5 \pm \sqrt{1}}{2} \leftarrow 25 - 24$$

$$\frac{5+\sqrt{1}}{2} \qquad \frac{5-\sqrt{1}}{2}$$

$$\frac{5+1}{2} \qquad \frac{5-1}{2}$$

$$3 \qquad 2$$

$\{2, 3\}$

---

b) $-2x^2 + 8 - 4x = 0$

Divide out a -2 this reduces simplifying at the end.

$$\frac{-2x^2}{-2} - \frac{4x}{-2} + \frac{8}{-2} = \frac{0}{-2}$$

$x^2 + 2x - 4 = 0$

$a = 1 \quad b = 2 \quad c = -4$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$\sqrt{20}$

$2 \cdot 10$

$2 \cdot 5$

$$x = \frac{-2 \pm \sqrt{20}}{2}$$

$$\frac{-2+\sqrt{20}}{2} \rightarrow \frac{-2+2\sqrt{5}}{2} = -1+1\sqrt{5}$$

$$\frac{-2-\sqrt{20}}{2} \rightarrow \frac{-2-2\sqrt{5}}{2} = -1-1\sqrt{5}$$

$\{-1 \pm 1\sqrt{5}\}$

c) $3x^2 = -1$

$\pm 1 \quad \pm 1$

$3x^2 + 1 = 0$

$a = 3 \quad b = 0 \quad c = 1$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(3)(1)}}{2(3)}$$

$\sqrt{-12} \rightarrow 2i\sqrt{3}$

$\sqrt{-1} \quad \sqrt{12}$

i

$12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3$

$$x = \frac{\pm\sqrt{-12}}{6}$$

$$\left\{\frac{\pm i\sqrt{3}}{3}\right\}$$

$$+\frac{2i\sqrt{3}}{6} \qquad -\frac{2i\sqrt{3}}{6}$$

$$\frac{i\sqrt{3}}{3} \qquad -\frac{i\sqrt{3}}{3}$$

d) $2x^2 + 4x - 5 = 2$

$-2 \quad -2$

$2x^2 + 4x - 7 = 0$

$a = 2 \quad b = 4 \quad c = -7$

$$\left\{\frac{-2 \pm 3\sqrt{2}}{2}\right\}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)}$$

$72 = 6\sqrt{2}$

$72 = 2 \cdot 36 = 2 \cdot 2 \cdot 18 = 2 \cdot 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$$x = \frac{-4 \pm \sqrt{72}}{4}$$

$$\frac{-4 + 6\sqrt{2}}{4} = \frac{-2 + 3\sqrt{2}}{2}$$

$$\frac{-4 - 6\sqrt{2}}{4} \qquad \frac{-2 - 3\sqrt{2}}{2}$$