2.3 – Exponential and Logarithmic Functions

An **Asymptote** is a vertical ($x = \#$) or horizontal ($y = \#$) line that a function's graph will NOT cross. *An asymptote is represented by a dotted line on a graph. ($y=1$, $x=-2$)

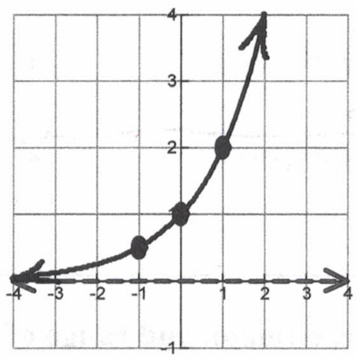
Exponential / Logarithmic Functions and Their Characteristics

➢ **Exponential function** → a function in the form $y=(b)^x$ where $b > 0$, $b \neq 1$, and x is IR. (All real #'s)

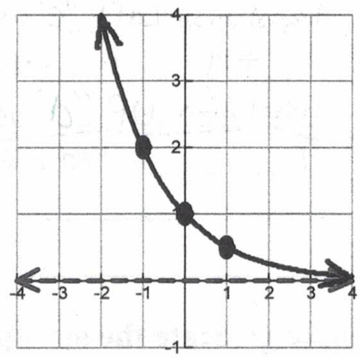
Exponential Function's Characteristics	
Domain: $(-\infty, \infty)$	Range: $(0, \infty)$ AKA (HA, ∞)
Common Pt: $(0, 1)$	Horizontal Asymptote: HA $y = 0$
Note: If graph has a vertical transformation ($\pm d$) or Reflection $(-a)$ over x-axis, then the RANGE AND H.A # are effected.	

Graphs of Exponential Functions

a.) Graph of $y = (2)^x$



b.) Graph of $y = (\frac{1}{2})^x$



Inverses of EACH OTHER

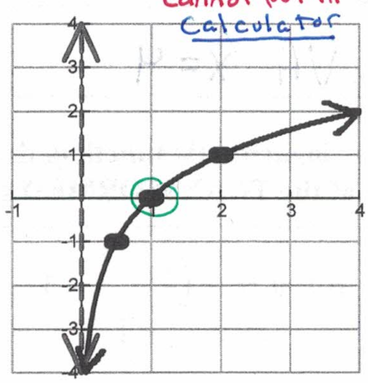
➢ **Logarithmic function** → a function in the form $y = \log_b(x)$ where $b > 0$, $b \neq 1$, and x ~~is IR~~ >0.

Logarithmic Function's Characteristics	
Domain: $(0, \infty)$ AKA (VA, ∞)	Range: $(-\infty, \infty)$
Common Pt: $(1, 0)$	Vertical Asymptote: VA $x = 0$
Note: If graph has a horizontal transformation or Reflection over y-axis, then the Domain AND VA # are effected.	

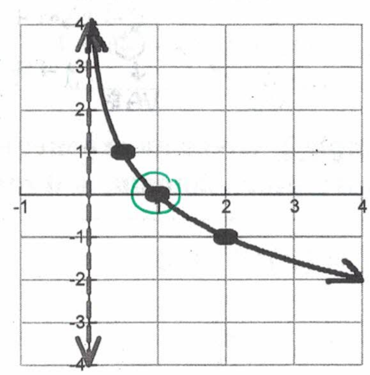
Graphs of Exponential Functions

a.) Graph of $y = \log_2(x)$

cannot put in calculator



b.) Graph of $y = \log_{1/2}(x)$



❖ If base > 1, the graph is increasing or "growing".

❖ If $0 <$ base < 1, the graph is decreasing or "decaying".

❖ A MAJOR characteristic between exponential and logarithmic functions is that they are INVERSES of each other.

❖ 10^x (base 10) and $\log x$ (common log) are inverses of each other $\log x \longleftrightarrow \log_{10} x$

❖ e^x (natural base) and $\ln x$ (natural log) are inverses of each other

$e^1 \approx 2.71828$

Transforming Exponential & Logarithmic Graphs

$y = a \cdot (base)^{bx \pm c} \pm d$ or $y = a \cdot \log_{base}(bx \pm c) \pm d$

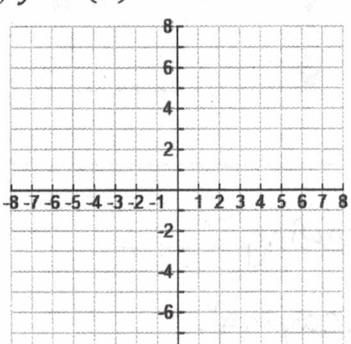
- If $a < 0$ then graph will have a reflection over the x-axis.
- If $a > 1$ then graph will have a vertical stretch by "a".
- If $+c$ then graph will have a translation Left "c" units.
- If $+d$ then graph will have a translation UP "d" units.
- If ________ then graph will have a ________.
- If ////// then graph will have a ///////.
- If $0 < a < 1$ then graph will have a vertical compression by "a".
- If $-c$ then graph will have a translation Right "c" units
- If $-d$ then graph will have a translation down "d" units.
- If ________ then graph will have a ________.

Example 1: State the asymptote, domain, and range of each given function using interval notation.

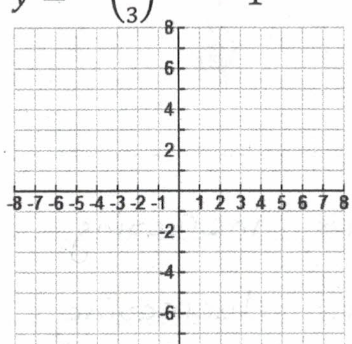
Given Exp / Log Function	Asymptote	Domain	Range
Exp a.) $f(x) = 4^{x-3} + 5$ (base; c → right 3 units; d ↑ up 5 units → HA #)	HA $y = 5$	$(-\infty, \infty)$	$(5, \infty)$ (HA, ∞)
Log b.) $f(x) = \log_3(x + 4) - 3$ (base; c → left 4 → VA; d down 3)	VA $x = -4$	$(-4, \infty)$ (VA, ∞)	$(-\infty, \infty)$
Exp c.) $f(x) = (1/3)^{x+5} - 2$ (base; c → left 5; d → down 2 units → HA #)	HA $y = -2$	$(-\infty, \infty)$	$(-2, \infty)$
Natural Log d.) $f(x) = \ln(x - 4) + 1$ (natural log; c → right 4 → VA #; d → up 1)	VA $x = 4$	$(4, \infty)$	$(-\infty, \infty)$

Example 2: Given an exponential or logarithmic function, draw the parent AND transformed graph. State the asymptote, domain, and range of the TRANSFORMED graph on provided lines.

a.) $y = (2)^{x+1} + 2$



b.) $y = -\left(\frac{1}{3}\right)^{x-2} - 1$



c.) $y = 2\log_4(x + 3) - 3$