2.2 – Exponential Equations w/Same Base

An Exponential equation is an equation that has the variable as an exponent.

ie: $2^{x+4} = 16$ (base: 2; exponent: $x+4$)

Solving Exponential Equations

There are two types of exponential equations:

A) BOTH SIDES of the equation CAN (changed) be transformed into the SAME BASE.

ie: $3^{2x-1} = 3^5$ (SAME BASES)

OR $2^{x+5} = 16$ → can be changed to a base of 2! → $2^{x+5} = 2^4$ (SAME base)

Once powers have the SAME BASE, set the exponents equal and solve!

B) BOTH SIDES of the equation CAN NOT be transformed into the same base, must use Logarithms to solve.

ie: $2^{x+5} = 49$ ← cannot change to a base of 2.

For both types of EXPONENTTIAL EQUATIONS:

1.) Keep answers as REDUCED fractions when possible.

2.) If a decimal **CAN NOT** be turned into a fraction, **ROUND** to thousandths place (3 decimal places).

Type 1: Both sides of the equation have the SAME base, then set the EXPONENTS equal to each other and solve the equation.

Example 1: Solve each exponential equation and keep answers in simplified fractional form.

a.) $2^{2x+7} = 2^{1}$ (SAME BASE) ← It is implied that there is a 1 there!

$2x+7 = 1$

$-7 \quad -7$

$2x = -6$

$x = -3$

$\{-3\}$

b.) $4^{2x+3} = 4^{5x-9}$ (SAME base)

$2x+3 = 5x-9$

$-5x \quad -5x$

$-3x+3 = -9$

$-3 \quad -3$

$-3x = -12$

$x = 4$

$\{4\}$

c.) $3^{4-3x} = (3^{2})^{3}$ (SAME Base; Power Raised to a Power)

$4-3x = 2(3)$

$4-3x = 6$

$-4 \quad -4$

$-3x = 2$

$x = -2/3$

$\{-2/3\}$

d.) $7^{3x-8} = \frac{1}{7}$ (NOT SAME BASE)

$7^{3x-8} = \frac{1}{7^1}$

$7^{3x-8} = 7^{-1}$ (SAME BASE)

$3x-8 = -1$

$+8 \quad +8$

$3x = 7$

$x = 7/3$

$\{7/3\}$

Type 2: Both sides of the equation don't have the same bases but can be changed Into the same base using exponent properties. $\sqrt{x} \to \sqrt[2]{x} \to x^{1/2}$ $\sqrt[3]{x^5} \to x^{5/3}$

Example 2: Solve each exponential equation and keep answers in simplified fractional form.

a.) $2^{2x+5} = 8$ ← make into a base of 2!

NOT SAME BASE

$2^3 \to 8$

$2^{2x+5} = 2^3$

$2x+5 = 3$

$2x = -2$

$x = -1$

$\{-1\}$

b.) $3^{4-3x} = 9$

$3^{4-3x} = 3^2$

$4-3x = 2$

$-4 \quad -4$

$-3x = -2$

$x = 2/3$

$\{2/3\}$

c.) $4^{3x} = 32^{x+1}$

Both need to be changed to a base of 2.

$(2^2)^{3x} = (2^5)^{x+1}$ * Powers raised to Powers!

$2(3x) = 5(x+1)$

$6x = 5x + 5$

$x = 5$

$\{5\}$

e.) $8^{3x+1} = 32^{x+3}$

$(2^3)^{3x+1} = (2^5)^{x+3}$

$3(3x+1) = 5(x+3)$

$9x+3 = 5x+15$

$-5x \quad -5x$

$4x+3 = 15$

$4x = 12$

$x = 3$

$\{3\}$

f.) $6^{2x+6} = \left(\frac{1}{36}\right)^{x+4}$

$6^{2x+6} = \left(\frac{1}{6^2}\right)^{x+4}$

$6^{2x+6} = (6^{-2})^{x+4}$

$2x+6 = -2(x+4)$

$2x+6 = -2x-8$

$4x+6 = -8$

$4x = -14 \to x = -14/4$

$\{-7/2\}$

g.) $49^{3x-2} = \sqrt{7}$

$(7^2)^{3x-2} = 7^{1/2}$

$2(3x-2) = \frac{1}{2}$

$2(6x-4) = \frac{1}{2}(2)$

$12x-8 = 1$

$12x = 9$

$x = 9/12 \to 3/4$

$\{3/4\}$

Example 3: Complete each problem involving Exponential Equations with the SAME Base

a.) Solve for x: $\frac{1}{8} \cdot 4^{3x-1} = \frac{1}{4}^{2-3x}$

$\frac{1}{2^3} \cdot (2^2)^{3x-1} = \left(\frac{1}{2^2}\right)^{2-3x}$

$2^{-3} \cdot 2^{6x-2} = (2^{-2})^{2-3x}$

multiplying powers with same base, add exponents.

$-3+6x-2 = -2(2-3x)$

$-5+6x = -4+6x$

$-6x \quad -6x$

$-5 \neq -4$

$\emptyset$

b.) Given $5 \cdot 2^{2x-4} + 8 = 28$

Find: 6^{2-x}

① Solve for x on Given!

$5 \cdot 2^{2x-4} + 8 = 28$

$-8 \quad -8$

$\frac{5 \cdot 2^{2x-4}}{5} = \frac{20}{5}$

$2^{2x-4} = 4$

$2^{2x-4} = 2^2$

$2x-4 = 2$

$2x = 6$

$x = 3$

Start solving the eqn.

② Find 6^{2-x}

6^{2-x}

$6^{2-(3)}$

$6^{-1} \to \frac{1}{6}$