2.1 – Basic Counting Concepts

Basic Counting Concept # 1 – The Fundamental Counting Principle

– **outcome** → the result of a single trial **Ex:** Flipping a coin (heads or tails)

– **sample space** → the set of all possible outcomes {heads, tails}

– **event** → one or more outcomes of a trial

- independent events – the outcome of one event does NOT EFFECT the outcome of another event
  w/ replacement
- dependent events – the outcome of one event DOES EFFECT the outcome of another event
  w/out replacement.

There are two ways to determine the possible outcomes for either events →

- Visually → create a diagram or a table which is particularly useful for independent events
- Mathematically → use the Fundamental Counting Principle which is useful for several multiple choices of independent events or various dependent events

Fundamental Counting Principle (FCP) →

If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$

Example 1: The following events are independent – Complete each problem using the FCP.

a.) A sandwich vendor offers a choice of hamburger, chicken, or fish on either a plain or sesame seed bun. How many different types of sandwiches are there to choose from?

3 protiens 2 buns

$3 \times 2 = 6$

H C F

S P S P S P

HS HP CS CP FS FP

b.) Kim won a contest on a radio station. The prize was a restaurant gift certificate to one of the city's three best restaurants and tickets to the following sporting events: football, baseball, basketball, or hockey. How many different ways can she select a prize?

$3 \cdot 4 = 12$ ways

Resturants Sports

c.) Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?

* # Pad has #'s 0–9 (10 digits)

1st digit	2nd digit	3rd digit
10 ·	10 ·	10

1000 codes

Example 2: The following events are dependent – Complete each problem using the FCP.

a.) Charlene wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?

Pd	1	2	3	4	5	6
# of Choices	6	5	4	3	2	1

FCP = $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$6! = 720$ schedules

b.) A computer's 6 character password can be formed if the first two characters are letters and the remaining characters are digits where both types of character can't be repeated. How many possible passwords could there be?

Characters	1st	2nd	3rd	4th	5th	6th
Choices	26	25	10	9	8	7

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

3,276,000 Passwords

c.) How many different 5-digit codes are possible (referring to a key pad) if the first digit can not be 0 and rest of the digits after the first can be used more than once?

Characters	1st	2nd	3rd	4th	5th
Choices	9	10	10	10	10

$9 \cdot 10^4 = 90{,}000$ codes

Factorial (!)

$4! \to 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Calculator

Factorial is under $\boxed{\text{MATH}}$ → PRB

12! → 12 $\boxed{\text{MATH}}$ → PRB #4: ! $\boxed{\text{ENTER}}$

$\frac{8!}{4!} = 1680$ $\quad \frac{8!}{4!} \to \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

Basic Counting Concept # 2 – Permutations and Combinations

– **permutation** → a group of objects or people arranged in a CERTAIN ORDER. ORDER MATTERS!

- Words that indicate a permutation – arrange or place / placements.
- Simple Example – A softball coach is arranging a list of 7 possible players of the team's 16 players for the team's batting lineup. How many arrangements can the coach have on his list? $_{16}P_7$

Computing a Permutation With a Calculator (with no repetitions) → **Use notation:** $_nP_r$ (total # (big), taking # (small))

1.) type in "n" first (always will be the bigger # of the two)
2.) MATH , scroll to PRB , select # 2 (nPr)
3.) type in "r" (always will be the smaller # of the two)

$_nP_r = \frac{n!}{(n-r)!}$

Computing a Permutation With a Formula (with repetitions) → *Cannot use Calculator*

The number of permutations of n objects of which p are alike and q are alike is given by $\frac{n!}{(p!q!)}$ (n! total #; repeating #'s)

– **combination** → a group of objects or people arranged NOT in a certain order!

- Words that indicate a combination – select or choose
- Simple Example – The Smith family is choosing a pizza for dinner and must choose 2 toppings out of a list of 6. How many different types of pizza can they have to choose from? $_6C_2 = 15$ types

Computing a Combination With a Calculator → **Use the notation:** $_nC_r$ (taking #; total #)

1.) type in "n" first (always will be the bigger # of the two)
2.) MATH , scroll to PRB , select # 3 (nCr)
3.) type in "r" (always will be the smaller # of the two)

$_nC_r = \frac{n!}{[(n-r)!]r!}$

Example 3: Determine if PERMUTATION or COMBINATION then complete problem.

a.) There are 10 finalists in a figure skating contest. How many ways can gold, silver, and bronze medals are awarded? Permutation nPr n = 10 r = 3 $_{10}P_3 = 720$ ways	b.) Jeremy is selecting three of fifteen flavors of ice cream. How many possibilities are there? combination nCr n = 15 r = 3 $_{15}C_3 = 455$	c.) A teacher divides his class into eight groups for a project. He will choose four groups to present their projects. In how many different ways can he choose the presentations? Combination $_8C_4 = 70$ ways	d.) How many different ways can five members of a club's ranked committee that has nine members be placed on a stage? Permutation $_9P_5 = 15,120$ ways
e.) How many different ways can the word MISSISSIPPI be arranged? Permutation w/ repeating n = 11 p = 2 i = 4 s = 4 $P = \frac{11!}{(2!4!4!)}$ 34,650 ways	f.) The manager of a four-screen movie theater is deciding which of 12 available movies to show. How many ways can he arrange the movies on the screens? Permutation $_{12}P_4 = 11,880$ ways	g.) Abby is trying to choose nine books out of a list of twelve from a reading list. How many options can she have? Combination n = 12 r = 9 $_{12}C_9 = 220$ ways	h.) How many different ways can Johnny place an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf? Permutation n = 5 r = 5 $_5P_5 = 120$ ways
i.) You will draw winners from a total of 25 tickets in a raffle. The first ticket wins \$100. The second ticket wins \$50. The third ticket wins \$10. In how many different ways can you draw the three winning tickets? Permutation	j.) How many different ways can the word COMPANY be arranged? Permutation NO LETTERS REPEAT n = 7 r = 7 $_7P_7 = 5040$ ways	k.) You have 20 songs on your iPhone. You have time to listen to three of the songs. In how many ways can you choose the three songs? Combination n = 20 r = 3 $_{20}C_3 = 1140$ ways	l.) A principal is starting a mentoring group. He needs to narrow his choice of students to six from a group of nine. How many ways can he choose a group? Combination n = 9 r = 6 $_9C_6 = 84$ ways