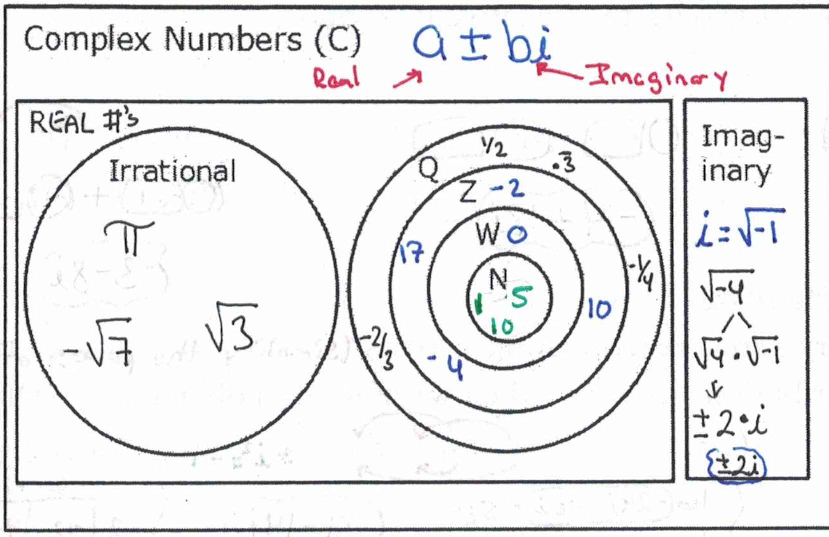
Number System Review

Complex Numbers Euler Diagram:



Q: Rational #'s: 1/2, −1/3, 8, −9
Z: Integers: …, −2, −1, 0, 1, 2, …
Whole #'s: 0, 1, 2, 3, …
Natural #'s: 1, 2, 3, 4, …
(counting #'s)

Imaginary Numbers

A number whose square is less than zero (negative)

*i is not a variable, it is a symbol that has a value of $\sqrt{-1}$.

Imaginary number $\sqrt{-1}$ is called "i" $\sqrt{-1}=i$

Other imaginary numbers – write using "i" notation: $\sqrt{-16}$ = $\pm 4i$ $\sqrt{-8}$ = $\pm 2i\sqrt{2}$

$\sqrt{16}\cdot\sqrt{-1} \rightarrow \pm 4\cdot i$

$\sqrt{8}\cdot\sqrt{-1}$, $2\sqrt{2}\cdot i$

Adding or subtracting imaginary numbers: add coefficients, just like monomials

- Add: $5i + 3i$ = $8i$

$i^2 \rightarrow i\cdot i \rightarrow \sqrt{-1}\cdot\sqrt{-1} \rightarrow -1$

Multiplying i:

$i^3 \rightarrow i\cdot i\cdot i \rightarrow \sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{-1} = -1\cdot\sqrt{-1} = -1(i) = -i$

$i^2\cdot i = -1\cdot i = -i$

$\sqrt{x}\cdot\sqrt{x} = x$
$\sqrt{5}\cdot\sqrt{5} = 5$
$\sqrt{3x}\cdot\sqrt{3x} = 3x$
$\sqrt{-2}\cdot\sqrt{-2} = -2$

- i^0 = 1 i^1 = i i^2 = −1 i^3 = $-i$
- i^4 = 1 i^5 = i i^6 = −1 i^7 = $-i$
- i^8 = 1 i^9 = i i^{10} = −1 i^{11} = $-i$
- Is there a pattern? YES!
- Apply the pattern to find: i^{17} = i i^{26} = −1

$i^{17} \rightarrow 4\overline{)17}$ = 4 r 1, −16, 1 $\rightarrow (i^4)^4 \rightarrow (1)^4 = 1 \cdot i^1 = i$

i^{26}: $4\overline{)26}$ = 6, −24, 2

Add up to 26: $i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^2$ → $1 \cdot (-1) = -1$

Complex Numbers

Take the form a + bi, where a and b are real numbers, and i is the imaginary number $\sqrt{-1}$

a + bi → Complex Number!
real imaginary

a is the real part of the number, and bi is the imaginary part of the number

When a = 0, we have a pure imaginary number

Given 2 + 5i, the real part is 2 and the imaginary part is 5i

complex #

Adding and Subtracting Complex Numbers

Combine like terms

Add real parts, add imaginary parts

When subtracting, distribute the negative then add

Simplify:

$(8 - 3i) + (2 + 5i)$ $10 + 2i$

$(8 + 7i) + (-12 + 11i)$ $-4 + 18i$

$(9 - 6i) - (12 + 2i)$

$(9 - 6i) + (-12 - 2i)$

$-3 - 8i$

Multiplying Complex Numbers

Multiply imaginary parts, remembering i^n rules (simplify the powers of i)

Use multiple distribution, just like when we multiply polynomials (FOIL)

$\sqrt{-8}\cdot\sqrt{-8} \neq \sqrt{64}$

$5(\sqrt{-2\cdot 4})(\sqrt{-8})$

$5(\sqrt{-8})(\sqrt{-8})$

$5(-8)$

-40

$(8 + 5i)(2 - 3i)$ $* i^2 = -1$

$16 - 24i + 10i - 15i^2$

$16 - 14i - 15i^2$

$16 - 14i - 15(-1) \longrightarrow 16 - 14i + 15$

$31 - 14i$

$(-6 + 2i)(2 - 3i)$

	-6	$+2i$
2	-12	$4i$
$-3i$	$18i$	$-6i^2$

$\rightarrow -12 + 22i - 6i^2$

$-12 + 22i - 6(-1)$

$-6 + 22i$

Complex Numbers: Division and Complex Conjugates

conjugates have OPPOSITE operation symbols

The complex conjugate of $a + bi$ is $a - bi$ $a \pm bi$

Multiply complex conjugates: $(a + bi)(a - bi)$ $a^2 - abi + abi - b^2i^2 \rightarrow a^2 - b^2i^2 \rightarrow a^2 + b^2$

$\hookrightarrow i^2 = -1$

Use conjugates to rationalize complex denominators of fractions

- Multiply numerator and denominator by conjugate of denominator
- Examples:

$\frac{5 + 2i}{7 - 4i} \cdot \frac{7 + 4i}{7 + 4i}$

$\frac{35 + 20i + 14i + 8i^2}{49 - 16i^2}$

$\frac{35 + 34i + 8i^2}{49 - 16(-1)} \rightarrow \frac{35 + 34i - 8}{49 + 16}$

$\frac{27 + 34i}{65}$

$\frac{8 + i}{2 - i} \cdot \frac{2 + i}{2 + i}$

$\frac{16 + 8i + 2i + i^2}{4 - i^2}$

$\frac{16 + 10i + i^2}{4 - (-1)} \longrightarrow \frac{16 + 10i - 1}{5}$

$\frac{15 + 10i}{5}$

$\frac{3 + 2i}{1}$

$\frac{3 - 2i}{3 + 2i} \cdot \frac{3 - 2i}{3 - 2i}$

$\frac{9 - 6i - 6i + 4i^2}{9 - 4i^2}$

$\frac{9 - 12i + 4i^2}{9 - 4(-1)} \rightarrow \frac{9 - 12i + 4(-1)}{13}$

$\frac{9 - 12i - 4}{13}$

$\frac{5 - 12i}{13}$

Finding Complex Solutions

Solve: $4x^2 + 100 = 0$ __________________ $3x^2 + 48 = 0$ __________________