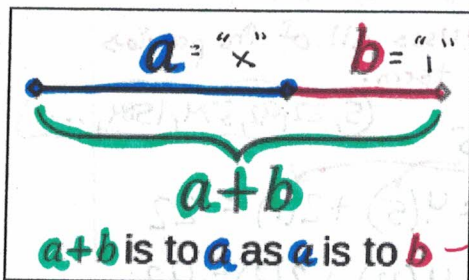


1.7 – Recursive and Special Sequences

Specific Sequence # 3 – Fibonacci Sequence

The following terms are in the Fibonacci Sequence → 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ? ...

- What is the pattern to this sequence? The sum of the two PREVIOUS terms
- What is the missing term (value of the ?) to this sequence? 55 + 89 = 144
- The Golden Ratio is a phenomena that is associated with the Fibonacci Sequence →



The Golden Ratio actually equals a specific number, let's figure it out:

SET UP AS A
Proportion!

$$\frac{a+b}{a} = \frac{a}{b} \rightarrow \frac{x+1}{x} = \frac{x}{1}$$

Solve using Quadratic Formula!
 $a=1 \quad b=-1 \quad c=-1$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$$x \approx 1.618033989$$

* phi (ϕ)

* **Example 1:** Create a collage of at least 10 pictures demonstrating Fibonacci Sequence/Golden Ratio that occur naturally in the human world. Refer to your "Cut-out Sheet" to create your collage.

Complete on your own
at home before 1st
Notebook ✓

Specific Sequence # 4 – Recursive Sequence (Formula)

– recursive sequence → a sequence that uses a previous term to get to the next term

where the sequence is neither consistently adding/subtracting (like arithmetic sequences)

nor is the sequence consistently multiplying/dividing (like geometric sequences)

Note: The following are examples of recursive sequences → $a_n = 2a_{n-1} + 5$ or $a_{n+1} = 2a_n + 5$

Example 2: Find the first five terms of the given sequence. Write your answer as a sequence.

<p>a.) $a_{n+1} = 3a_n - 2$; $a_1 = 4$</p> <p>$a_1 = 4$ $(4, 10, 28, 82, 244, \dots)$</p> <p>$a_2 = 3(4) - 2 = 10$ $a_1 \rightarrow$</p> <p>$a_3 = 3(10) - 2 = 28$ $a_2 \rightarrow$</p> <p>$a_4 = 3(28) - 2 = 82$</p> <p>$a_5 = 3(82) - 2 = 244$</p>	<p>b.) $a_n = -2(a_{n-1} + 6)$; $a_1 = 3$</p> <p>$a_1 = 3$ $(3, -18, 24, -60, 108, \dots)$</p> <p>$a_2 = -2(3 + 6) = -18$ $a_{2-1} = a_1$</p> <p>$a_3 = -2(-18 + 6) = 24$ $a_{3-1} = a_2$</p> <p>$a_4 = -2(24 + 6) = -60$ $a_{4-1} = a_3$</p> <p>$a_5 = -2(-60 + 6) = 108$ $a_{5-1} = a_4$</p>	<p>c.) $a_{n+1} = 4a_n + 2n$; $a_1 = 5$</p> <p>* Uses all of the previous term.</p> <p>$a_1 = 5$ $(5, 22, 92, 374, 1504, \dots)$</p> <p>$a_2 = 4(5) + 2(1) = 22$</p> <p>$a_3 = 4(22) + 2(2) = 92$</p> <p>$a_4 = 4(92) + 2(3) = 374$</p> <p>$a_5 = 4(374) + 2(4) = 1504$</p>
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Example 3: Complete the word problem below using recursive sequences.

Caleb discovered that there were 225 dandelions in his garden on the first Saturday of spring. He had time to pull out 100, but by the next Saturday, there were twice as many as he had left. Each Saturday in spring, he removed 100 dandelions, only to find that the number of remaining dandelions had doubled by the following Saturday.

<p>a.) Write a recursive formula for the number of dandelions Caleb finds in his garden each Saturday.</p> <p>$d_n = \#$ of dandelions at begin of "n^{th}" Sunday</p> <p>Caleb pulls out 100 → $d_n - 100$</p> <p>$d_{n+1} = \#$ of dandelions of the following Saturday but will be TWICE AS MUCH!</p> <p>$d_{n+1} = 2(d_n - 100)$</p>	<p>b.) Find the number of dandelions Caleb would find on the fourth Saturday.</p> <p>$d_{n+1} = 2d_n - 200$</p> <p>$d_1 = 225$</p> <p>$d_2 = 2(225) - 200 = 250$</p> <p>$d_3 = 2(250) - 200 = 300$ $d_4 = 2(300) - 200 = 400$</p> <p>400 dandelions remain!</p>
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– iteration → the process of composing a function with ITSELF repeatedly

Example 4: Find the first three terms x_1 , x_2 , and x_3 of each function.

<p>a.) $f(x) = 2x + 3$ where $x_0 = 1$</p> <p>$x_1 = f(x_0)$ $x_2 = f(x_1)$ $x_3 = f(x_2)$</p> <p>$x_1 = f(1)$ $x_2 = f(5)$ $x_3 = f(13)$</p> <p>$x_1 = 2(1) + 3$ $x_2 = 2(5) + 3$ $x_3 = 2(13) + 3$</p> <p>$x_1 = 5$ $x_2 = 13$ $x_3 = 29$</p>	<p>b.) $f(x) = 6 + 4x - x^2$ where $x_0 = 3$</p> <p>$x_1 = f(x_0)$ $x_2 = f(x_1)$ $x_3 = f(x_2)$</p> <p>$x_1 = 6 + 4(0) - (0)^2$ $x_2 = 6 + 4(4) - (4)^2$ $x_2 = 6 + 4(-39) - (-39)^2$</p> <p>$x_1 = 6$ $x_2 = -39$ $x_3 = -1671$</p> <p>$x_1 = 9$ $x_2 = -39$ $x_3 = -1671$</p>
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