1.6 – Geometric Sequences and Series (F and I) Word Problems

Use the following formulas for the word problems below:

$a_n = a_1(r)^{n-1}$ sequence $\qquad S_n = \dfrac{a_1(1-r^n)}{(1-r)}$ Finite Series $\qquad S = \dfrac{a_1}{(1-r)}$ Infinite Series

I. In the following set of examples, assume all geometric series are finite.

1.) A one-ton ice sculpture is melting at a rate of 80% of its original weight per hour. How much of the sculpture, in pounds, will be left when standing inside for 6 hours? Sequence

$a_1 = 1$ ton. $r = 80\% \to .8$

2000 lbs $a_6 = ?$

After 6hrs it weighs 655.36 lbs

2.) A ball is dropped at specific height, in feet, and then rebounds. On the eighth bounce, the ball's height is 0.02 feet. On the fifth bounce, the ball's height is 0.92 feet. What percent is the ball rebounding from its original height? Sequence

$r = ?$ %

$a_5 = .92$ $a_8 = .02$

gap

27.89%

3.) Benny got a new job that guarantees him a raise every year. His annual pay raise is always 4% of his salary at that time. After four years at the job, Benny's salary is $53,814. What was Benny's starting salary when he was hired? Sequence

$a_1 = ?$ $a_5 = 53{,}814$

$r = 100\% + 4\%$

104%

$r = 1.04$

Starting salary was $46,000

4.) A culture of bacteria initially has 5,000 bacteria and its sizes ~~increases~~ decreases by 8% every hour. How many hours will it take for 2048 bacteria to remain in the culture?

Sequence $a_n = 2048$

$a_1 = 5000$ $r = 100\% - 8\%$

$r = 92\% \to .92$

After 12 hrs

5.) A ball is dropped from a height of 22 feet. Each time it drops, it rebounds 40% of the height from which it is falling. What is the total distance traveled in 18 bounces? Series

51.3 ft

6.) Heavy rain caused a river to rise. The river rose three inches the first day, then it rose six inches the second day, and so on. How many days did it take for the river to rise 765 inches? Series

$a_1 = 3$ $a_2 = 6$ $r = 2$

$S_n = 765$ $n = ?$

8 days

II. In the following set of examples, assume all geometric series are infinite.

7.) A child on a swing is given a big push. She travels 12 feet on the first swing but only $\frac{5}{6}$ as far on each successive swing. How far (total distance) does she travel before the swing stops? Series

$a_1 = 12$ $r = 5/6$

72 ft

8.) A ball is thrown 12 meters in the air. The ball rebounds a percent of the distance it falls. If the ball's total vertical distance is 480 meters, what percent is its rebound?

Series $a_1 = 24$

$r = ?$ $S = 480$

95% rebound

9.) Jack is working on a math problem on his homework and needs help. This is the problem he needs help on: $0.\overline{4} + 2.\overline{351}$. What answer (as a fraction) should Jack put on his paper?

1384/495

* SEE ADDITIONAL SHEETS FOR THE WORK FOR EACH EXAMPLE! *

1.6 Notes

① $a_n = a_1(r)^{n-1}$

$a_6 = 2000(.8)^{6-1}$

$a_6 = 2000(.8)^5$

$a_6 = 655.36$ lbs

② $a_n = a_1(r)^{n-1}$

$.02 = a_1(r)^{8-1} \rightarrow$ $\quad$ $.02 = \cancel{a_1} r^7$ divide

$.92 = a_1(r)^{5-1} \rightarrow$ $\quad$ $.92 = \cancel{a_1} r^4$

$.0217 = r^3$

$\sqrt[3]{.0217} = \sqrt[3]{r^3}$

$r = .2789$

$r = 27.89\%$

③ $a_n = a_1(r)^{n-1}$

$53{,}814 = a_1(1.04)^{5-1}$

$53{,}814 = a_1(1.04)^4$

$\frac{53{,}814}{(1.04)^4} = a_1$

$a_1 \approx \$46000.43$

④ $a_n = a_1(r)^{n-1}$

$\frac{2048}{5000} = \frac{5000}{5000}(.92)^{n-1}$

$.4096 = (.92)^{n-1}$

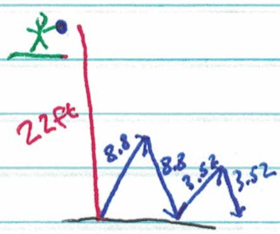
$\frac{\log .4096}{\log .92} = \frac{(n-1)\log .92}{\log .92}$

$10.7047 = n-1$

$n = 1\phi.7047$

$n = 12$ hrs

⑤



$a_1 = 22 \cdot .4$

$a_1 = 8.8$

$n = 18$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

Ball bounces tha to a height then drops that same height.

$22 + 2\left[\frac{a_1(1-r^n)}{(1-r)}\right]$

initial drop height

$22 + 2\left[\frac{8.8(1-.4^{18})}{1-.4}\right]$

$22 + 2\left[14.66666566\right]$

$S_{18} = 51.3$ ft

(6) $S_n = \dfrac{a_1(1-r^n)}{(1-r)}$

$765 = \dfrac{3(1-2^n)}{(1-2)}$

$765 = \dfrac{3(1-2^n)}{-1}$

$\dfrac{765}{-3} = \dfrac{-3(1-2^n)}{-3}$

$-255 = 1-2^n$

$\dfrac{-256}{-1} = \dfrac{-2^n}{-1}$

$256 = 2^n$

$\dfrac{\log 256}{\log 2} = \dfrac{n\log 2}{\log 2}$

$n = 8$

1.6 Notes (continued)

Ex. 2 Infinite Series

(7) $S = \frac{a_1}{(1-r)}$

$S = \frac{12}{(1-5/6)}$

$S = 72$ ft

(8) $S = \frac{a_1}{(1-r)}$

$\frac{480}{1} = \frac{24}{1-r}$

$480(1-r) = 24$

$480 - 480r = 24$

$-480r = -456$

$r = .95$

95%

(9) $0.\overline{4} + 2.3\overline{51}$

$$\begin{array}{r} 0.44444444\ldots \\ +\ 2.351515151\ldots \\ \hline 2.7959595\ldots \end{array}$$

$2.7 + [.095 + .00095 + .0000095 + \ldots]$

$a_1 = .095$ $a_2 = .00095$ $a_3 = .0000095$

$r = .01$

$S = \left[\frac{.095}{1-.01}\right] + 2.7$ ← non-repeating part!

$S = \frac{1384}{495}$