1.7 – Inverse Functions

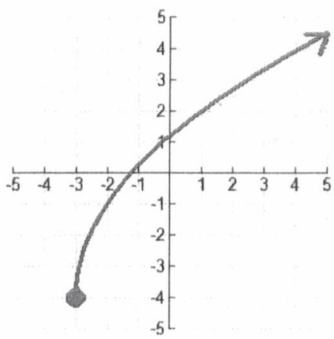
– **one-to-one function** → a function where every x has a unique y and every y has a unique x.

- Must pass the Vertical Line Test (VLT) and Horizontal Line Test (HLT).

* Vertical Line Test (VLT) → NO vertical line intersects graph more than once.
* Horizontal Line Test (HLT) → NO horizontal line intersects graph more than once.

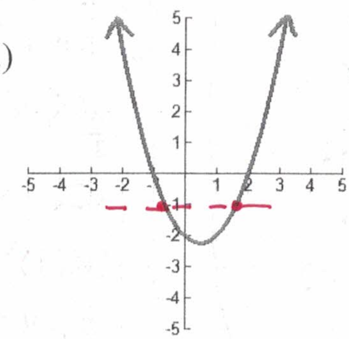
Example 1: Determine if the following functions are 1 – 1. State YES or NO.

a.)



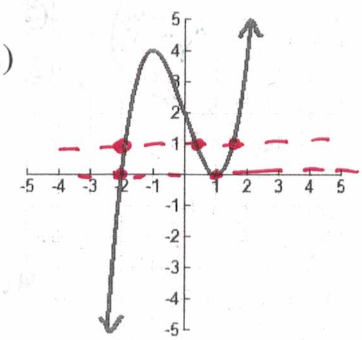
VLT → pass fail
HLT → pass fail
1 – 1? YES

b.)



VLT → pass fail
HLT → pass fail
1 – 1? NO

c.)



VLT → pass fail
HLT → pass fail
1 – 1? NO

– **inverse function** → an one-to-one function where if have f (**x**) = **y** then its inverse is $f^{-1}(y) = x$

* x and y switch place

- If a function contains point (x, y) than its inverse contains point (y, x).
- Domain of the function f (x) is the Range of the inverse $f^{-1}(x)$.
- Range of the function f(x) is the Domain of the inverse $f^{-1}(x)$.

Ex: If f (5) = 2, then f^{-1} (2) = 5 ; If f (x) = ½ x – 4 , then find f^{-1} (8) = 24

x y Inverse y x

$f(x) = \frac{1}{2}x - 4$ $f^{-1}(8)$
y y
$8 = \frac{1}{2}x - 4$
$12 = \frac{1}{2}x \rightarrow x = 24$

- Two functions are inverses if the following are TRUE: $f(g(x)) \rightarrow x$ AND $g(f(x)) \rightarrow x$

f(x) and g(x)

Example 2: Complete each problem about inverse functions.

a.) If $f(x) = 2 + 4\sqrt{x-5}$, then find $f^{-1}(14)$.

$f(x) = 2 + 4\sqrt{x-5}$
$14 = 2 + 4\sqrt{x-5}$
$-2 \quad -2$
$\frac{12}{4} = \frac{4\sqrt{x-5}}{4}$
$3 = \sqrt{x-5}$
$(3)^2 = (\sqrt{x-5})^2$
$9 = x - 5$
$x = 14$
$f^{-1}(14) = 14$

b.) If $f(x) = x^2 - 4x$ where $\boxed{x \geq 0}$, then find $f^{-1}(12)$.

domain restriction

$f(x) = x^2 - 4x$
$12 = x^2 - 4x$ (Set = to zero)
$x^2 - 4x - 12 = 0$
FACTOR!
$(x-6)(x+2) = 0$
ZPP
$x - 6 = 0 \quad x + 2 = 0$
$x = 6 \quad \cancel{x = -2}$ not ≥ 0
$f^{-1}(12) = 6$

c.) Are the two given functions inverses of each other?

f (x) = 4x – 5 and $g(x) = \frac{x+5}{4}$

f(g(x))	g(4x-5)
$f\left(\frac{x+5}{4}\right)$	$\frac{(4x-5)+5}{4}$
$4\left(\frac{x+5}{4}\right) - 5$	$\frac{4x-5+5}{4}$
$\cancel{4}\left(\frac{x+5}{\cancel{4}}\right) - 5$	$\frac{\cancel{4}x}{\cancel{4}}$
$x + 5 - 5$	x ✓
x ✓	

Yes, f(x) and g(x) are inverses!

Steps to Finding the Equation of the Inverse

* $f^{-1}(x)$ *

1.) Write f (x) as y 2.) Switch the x's and the y's 3.) Resolve for y (where $y = f^{-1}(x)$)

Example 3: Find the inverse of f (x). Your answer should be written as $f^{-1}(x) = ?$.

a.) $f(x) = 2x - 6$	b.) $f(x) = \frac{1}{4}x^2 + 3$; $x \leq 0$	c.) $f(x) = 3\sqrt{2x-4}$	d.) $f(x) = \frac{x+3}{x-2}$
① $y = 2x - 6$ ② $x = 2y - 6$ ③ Solve for "y" SAME $2y - 6 = x$ $+6 \quad +6$ $\frac{2y}{2} = \frac{x}{2} + \frac{6}{2}$ $y = \frac{1}{2}x + 3$ ④ Rewrite using $f^{-1}(x) =$ instead $y =$ $f^{-1}(x) = \frac{1}{2}x + 3$	domain Restriction ① $y = \frac{1}{4}x^2 + 3$ ② $x = \frac{1}{4}y^2 + 3$ ③ Solve for y $\frac{1}{4}y^2 + 3 = x$ $-3 \quad -3$ $\frac{1}{4}y^2 = x - 3$ $4(\frac{1}{4}y^2) = 4(x-3)$ $y^2 = 4x - 12$ $\sqrt{y^2} = \pm\sqrt{4x-12}$ $y = -\sqrt{4x-12}$ $f^{-1}(x) = -\sqrt{4x-12}$	① $y = 3\sqrt{2x-4}$ ② $x = 3\sqrt{2y-4}$ ③ Solve for y $\frac{3\sqrt{2y-4}}{3} = \frac{x}{3}$ $\sqrt{2y-4} = \frac{1}{3}x$ $(\sqrt{2y-4})^2 = (\frac{1}{3}x)^2$ $2y - 4 = \frac{1}{9}x^2$ $+4 \quad +4$ $\frac{2y}{2} = \frac{\frac{1}{9}x^2}{2} + \frac{4}{2}$ $y = \frac{1}{18}x^2 + 2$ $f^{-1}(x) = \frac{1}{18}x^2 + 2$	

Graphing the Inverse of a Function

1.) Find ordered pairs of f (x) (Use symmetry to determine which points to use). *Table on Calculator!*

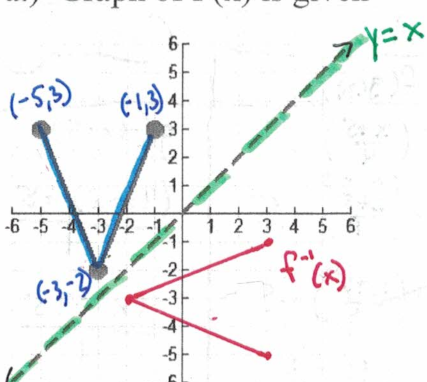
2.) Switch the x's and the y's within the ordered pairs found in Step 1.

3.) Plot the points and draw in the graph.

* The graph of a function and its inverse is Symmetric about the line y=x.

Example 4: Sketch the function f (x) and its inverse on the same graph. Dotted line is y = x.

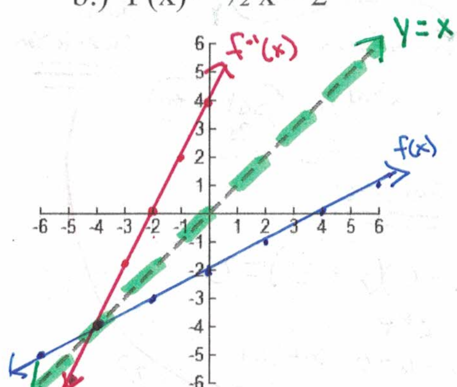
a.) Graph of f (x) is given



f(x)	
-5	3
-3	-2
-1	3

$f^{-1}(x)$	
3	-5
-2	-3
3	-1

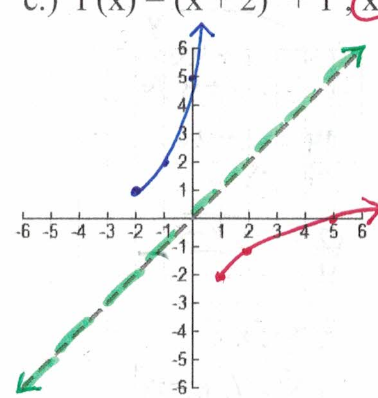
b.) $f(x) = \frac{1}{2}x - 2$



f(x)	
-6	-5
-4	-4
-2	-3
0	-2
2	-1
4	0

$f^{-1}(x)$	
-5	-6
-4	-4
-3	-2
-2	0
-1	2
0	4

c.) $f(x) = (x+2)^2 + 1$; $x \geq -2$ ↳ Domain Restriction



f(x)	
-2	1
-1	2
0	5
1	10

$f^{-1}(x)$	
1	-2
2	-1
5	0
10	1