1.5 – Geometric Series (Finite and Infinite)

$a_n = a_1(r)^{n-1}$

Specific Series # 2 – Finite Geometric Series

– **geometric series** → the indicated Sum of terms in a geometric sequence.

where it's represented by the following formula: $S_n = \dfrac{a_1(1-r^n)}{(1-r)}$

sum of the 1^{st} "n" terms — 1^{st} term — Common ratio — common ratio raised to the "n^{th}" power

* Reminder – Don't forget to put ( ) around any "r" that is a negative or a fraction

Example 1: Find the sum of each finite geometric series.

a.) $a_1 = 8,\ r = -3,\ \boxed{n = 7}$ Tell us that it is finite. $S_n = \dfrac{a_1(1-r^n)}{(1-r)}$ $S_7 = \dfrac{8(1-(-3)^7)}{(1-(-3))}$ $S_7 = \dfrac{8(1+2187)}{4}$ $S_7 = \dfrac{17504}{4}$ $S_7 = 4,376$	b.) $a_1 = 1{,}280,\ a_9 = 5$ Finite Step 1 Find r $5 = 1{,}280(r)^{9-1}$ $5 = 1{,}280r^8$ $\dfrac{1}{256} = r^8 \rightarrow \sqrt[8]{1/256} = \sqrt[8]{r^8}$ $\boxed{r = 1/2}$ Step 2 Find S_9 $S_9 = \dfrac{1280(1-(1/2)^9)}{(1-(1/2))}$ $S_9 = 2555$	c.) $4 + 24 + \ldots + 31{,}104$ $a_1\ \ a_2\ \ \ a_n$ $a_1 = 4\ \ r = 6\ \ a_n = 31{,}104$ Step 1 Find n $31{,}104 = 4(6)^{n-1}$ $7776 = 6^{n-1}$ $\dfrac{\log 7776}{\log 6} = \dfrac{(n-1)\log 6}{\log 6}$ $5 = n-1$ $n = 6$ Step 2 Find S_6 $S_6 = \dfrac{4(1-6^6)}{(1-6)}$ $S_6 = 37{,}324$	d.) $\displaystyle\sum_{n=4}^{18} 2(3)^{n-1}$ This indicates a finite series! Starts at 4 $a_1 = 2\ \ r = 3$ Step 1 Find a_4 $a_4 = 2(3)^{4-1} \rightarrow 54$ Step 2 Find S_{15} $n = 18-4+1$ $n = 15$ $S_{15} = \dfrac{54(1-3^{15})}{(1-3)}$ $S_{15} = 387{,}420{,}462$

Example 2: Use the finite geometric series formula to complete each problem.

a.) The $\boxed{\text{sum of first 8 terms}}$ is 39,360 and the common ratio is 3. $\boxed{\text{What is the first term?}}$ This indicates a finite series! $S_8 = 39{,}360\ \ n = 8\ \ r = 3\ \ a_1 = ?$ $39{,}360 = \dfrac{a_1(1-3^8)}{(1-3)}$ $(-2)\,39{,}360 = \dfrac{a_1(-6560)}{\cancel{-2}}\,\cancel{(-2)}$ $-78{,}720 = -6560a_1$ $a_1 = 12$	b.) The first term is – 3 and the common ratio is 4. $\boxed{\text{How many terms}}$ were added together to get a sum of –16,383? $a_1 = -3\ \ r = 4\ \ S_n = -16383\ \ n = ?$ $-16{,}383 = \dfrac{-3(1-4^n)}{(1-4)} \Rightarrow -16{,}383 = \dfrac{\cancel{-3}(1-4^n)}{\cancel{-3}}$ $-16{,}383 = 1 - 4^n$ $\ \ -1\ \ \ \ \ -1$ $\dfrac{-16{,}384}{-1} = \dfrac{-4^n}{-1}$ * can not take log of negative #. $16{,}384 = 4^n$ $\dfrac{\log 16{,}384}{\log 4} = \dfrac{n \log 4}{\log 4}$ $n = 7$ 7 terms

Specific Series # 3 – Infinite Geometric Series

– **infinite geometric series** → the indicated PARTIAL Sum of a geometric sequence.

where it's represented by the following formula: $S = \dfrac{a_1}{(1 - r)}$

- An infinite geometric series can do TWO THINGS:

1.) Converge → $-1 < r < 1$ where the Sum = # (merges to the #)

2.) Diverge → is not $-1 < r < 1$ where the sum DNE (splits)

Does Not Exist

Example 3: Determine if each series converges or diverges. If it's convergent, state the sum.

a.) $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \boxed{\ldots}$ indicates infinite

$\frac{\frac{1}{32}}{\frac{1}{8}}$ $\frac{\frac{1}{8}}{\frac{1}{2}}$ Converges

$r = 1/4$ $a_1 = 2$

$S = \dfrac{2}{(1 - 1/4)}$ converges to 8/3

$S = 8/3$

b.) $\frac{1}{2} + \frac{3}{2} + \frac{9}{2} + \frac{27}{2} + \ldots$

$a_1 = \frac{1}{2}$

$\frac{\frac{27}{2}}{\frac{9}{2}}$ $\frac{\frac{9}{2}}{\frac{3}{2}}$

$r = 3$

DNE b/c r is not between −1 and 1

c.) $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$ indicates a infinite series

$a_1 = 5$ $r = \frac{1}{2}$ Converges to 10

$S = \dfrac{5}{(1 - (1/2))}$

$S = 10$

converges to 10

d.) $\sum_{n=1}^{\infty} -2\left(\frac{4}{3}\right)^{n-1}$

$a_1 = -2$ $r = \frac{4}{3}$ or 1.33333333 1 1/3

$S =$ DNE

Diverges

Example 4: Use the infinite geometric series formula to complete each problem.

a.) The sum of an infinite geometric series is 81, and its common ratio is ⅔. What is the first term?

$S = 81$ $r = 2/3$ a_1

$81 = \dfrac{a_1}{(1 - 2/3)}$

$\dfrac{81}{1} = \dfrac{a_1}{1/3}$

$a_1 = 81(1/3)$

$a_1 = 27$

b.) The first term in an infinite geometric series is – 34, and its sum is – 42.5. What is common ratio?

$a_1 = -34$ $S = -42.5$ $r = ?$

$\dfrac{-42.5}{1} = \dfrac{-34}{1 - r}$

$-42.5(1 - r) = -34$

$-42.5 + 42.5r = -34$

$r = 1/5$

c.) Rewrite $1.\overline{42}$ as a series and then determine its fraction.