1.4 – Geometric Sequences

What is a ratio? - comparison of 2 #'s by division.

Specific Sequence # 2 – Geometric Sequence

– **geometric sequence** → a sequence where the ratio between any two consecutive terms is a constant, called r, the common ratio

Example 1: Complete each problem.

a.) Is the given sequence geometric? 3 , 9 , 27 , 81 , … If so, what is the value of r?	b.) Is the given sequence geometric? 96 , – 24 , 6 , – 1.5, … If so, what is the value of r?	c.) A geometric sequence has $a_1 = 4$ and r = 6. What is the fourth term of the sequence?
* To find "r", work backwards * $\frac{81}{27}$ $\frac{27}{9}$ $\frac{9}{3}$ 3 Yes, r = 3	$\frac{-1.5}{6}$ $\frac{6}{-24}$ $\frac{-24}{96}$ $-\frac{1}{4}$ Yes, r = $-1/4$	$a_1 = 4$ $a_2 = 4(6) = 24$ $a_3 = 24(6) = 144$ $a_4 = 144(6)$ $a_4 = 864$

"Nth Term Formula" of Geometric Sequence: Used to find ANY term of a geometric sequence

Consider a geometric sequence whose first term is a_1 and whose common ratio is r:

a_1	→	1st term (a_1)
$a_1 \cdot r$	→	2nd term ($a_2 =$ $a_1 \cdot r$)
$a_1 \cdot r \cdot r = a_1 \cdot r^2$	→	3rd term ($a_3 =$ $a_1 \cdot r^2$)
$a_1 \cdot r^2 \cdot r = a_1 \cdot r^3$	→	4th term ($a_4 =$ $a_1 \cdot r^3$)
$a_1 \cdot r^3 \cdot r = a_1 \cdot r^4$	→	5th term ($a_5 =$ $a_1 \cdot r^4$)

(General) nth term Formula: $a_n = a_1(r)^{n-1}$ → some important notes about this formula…

- formula will always be an Exponential Equation
- do not multiply a_1 # and r together to simplify the formula
- put ( ) around any "r" that's a negative or Fractional #

Example 2: Find what is indicated for each ~~arithmetic~~ geometric sequence.

a.) $a_1 = 2$ and r = 4, find the 8th term	b.) Find a_5 for the sequence $-1, \frac{1}{4}, -\frac{1}{16}, \ldots$	c.) Write the nth term formula (equation) for the sequence 3 , – 36 , 432 , …
$a_n = a_1(r)^{n-1}$ $a_8 = 2(4)^{8-1}$ $a_8 = 2(4)^7$ $a_8 = 32{,}768$	$a_1 = -1$ r = ? a_5 = ? Step 1 find r: $\frac{\frac{-1}{16}}{\frac{1}{4}}$ $\frac{\frac{1}{4}}{-1}$ r = $-1/4$ Step 2 find a_5: $a_5 = -1(-1/4)^{5-1}$ $a_5 = -1(-1/4)^4$ $a_5 = -1/256$	$a_1 = 3$ r = –12 $\frac{432}{-36}$ $\frac{-36}{3}$ –12 $a_n = 3(-12)^{n-1}$

Example 3: Considering all given sequences are geometric – Find what is asked.

a.) The 7th term of the sequence is 62,500 and the common ratio is 5. What is the first term?

$a_1 = ?$ $a_7 = 62{,}500$ $r = 5$

$62{,}500 = a_1(5)^{7-1}$

$62{,}500 = a_1(5)^6$

$\frac{62{,}500}{15{,}625} = \frac{15{,}625a_1}{15{,}625}$

$a_1 = 4$

b.) What is the common ratio for the sequence where the first term is 96 and the 6th term is 3?

$r = ?$ $a_1 = 96$ $a_6 = 3$ $a_n = a_1(r)^{n-1}$

$3 = 96(r)^{6-1}$

$\frac{3}{96} = \frac{96r^5}{96}$

$\frac{1}{32} = r^5 \rightarrow \sqrt[5]{1/32} = \sqrt[5]{r^5}$

$r = 1/2$

c.) Which term is 78,732 in the sequence of 4 , 12 , 36 , 108 , …?

$n = ?$ $a_n = 78{,}732$ $a_1 = 4$ $r = 3$

$\frac{78{,}732}{4} = \frac{4(3)^{n-1}}{4}$

$19{,}683 = 3^{n-1}$

In order to bring n-1 down, must take log of each side!

$\frac{\log 19{,}683}{\log 3} = \frac{(n-1)\log 3}{\log 3}$

$9 = n-1$

$n = 10$

10th term

d.) The fifth term in the sequence is 768 and the ninth term is 196,608. What is the third term of the sequence? $a_3 = ?$ $a_5 = 768$ $a_9 = 196{,}608$

Step 1 Find r

$196{,}608 = a_1 r^{9-1}$

$768 = a_1 r^{5-1}$

$\frac{196{,}608 = a_1 r^8}{768 = a_1 r^4}$ → Divide to 8 by bottom!

$256 = r^4$

$\sqrt[4]{256} = \sqrt[4]{r^4}$

$r = 4$

Step 2 find a_1 → $768 = a_1(4)^{5-1}$

$768 = a_1(4)^4$

$768 = 256a_1$

$a_1 = 3$

Step 3 Find a_3

$a_3 = 3(4)^{3-1}$

$a_3 = 48$

– **geometric means** → represent the terms between any 2 nonconsecutive terms of a geometric sequence

Ex: Circle the 4 geometric means between 2 and 468: 2 , (6) , (18) , (54) , (162) , 468 , …

Example 4: Complete each problem. Assume both sequences are geometric.

a.) Find the three geometric means for -6 , (−18) , (−54) , (−162) , -486

a_1 a_2 a_3 a_4 a_5

Step 1 Find r

$-486 = (-6)(r)^{5-1}$

$\frac{-486}{-6} = \frac{-6r^4}{-6}$

$81 = r^4$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$r = 3$

Step 2 find $a_2 - a_4$

$a_1 = -6$

$a_2 = -6(3) = -18$

$a_3 = -18(3) = -54$

$a_4 = -54(3) = -162$

b.) Find the two geometric means between -20 and 1280.

-20 , (80) , (−320) , 1280

a_1 a_2 a_3 a_4

Step 1

$1280 = (-20)(r)^{4-1}$

$1280 = -20r^3$

$-64 = r^3$

$r = -4$

Step 2

$a_1 = -20$

$a_2 = -20(-4) = 80$

$a_3 = 80(-4) = -320$