1.3 – Arithmetic Sequences and Series Word Problems

Use the following formulas for the word problems below:

$a_n = a_1 + d\cdot(n-1)$ (sequence) $\quad S_n = \frac{n}{2}(a_1 + a_n)$ (series) $\quad S_n = \frac{n}{2}(2a_1 + d\cdot(n-1))$ (series)

1.) According to the National Education Association, teachers in the US earned an average of $21,700 per year in 1984. This amount increased by approximately $1,472 yearly. Determine the amount a US teacher earned in the year 2005.

Sequence → $a_n = a_1 + d(n-1)$

$a_1 = 21{,}700 \quad d = 1{,}472 \quad n = 22$

$a_n = ?$

2005 − 1984 = 21 + 1

In 2005, a teacher earns $52,612.

2.) A drive-in theater has spaces for 20 cars in the first parking row, 22 in the second, 24 in the third, and so on. If there are 34 rows in the theater, find the number of cars that can be parked. Series

$S_n = \frac{n}{2}(a_1 + a_n)$

$a_1 = 20 \quad a_2 = 22 \quad a_3 = 24 \quad n = 34$

$a_{34} = ? \quad d = 2$

Total cars parked is 1802.

3.) The purchase value of an office computer is $12,500. The value of the computer after 6 years is $1,850. What was the computer's annual depreciation? Sequence $a_n = a_1 + d(n-1)$

$a_1 = 12{,}500 \quad d = ? \quad n = 7$ ← takes into account 1st yr. (6 + 1)

$a_7 = 1850$

It depreciates by $1775 per year.

4.) An architect designs a small theater with 15 seats in the first row, 18 in the second, 21 in the third, and etc. If the theater ~~is~~ has a seating capacity of 204 and 8 rows, how many seats are in the last row?

Use either sequence or series.

$a_1 = 15$, $a_2 = 18$, $a_3 = 21$ } $d = 3$ $\quad n = 8 \quad S_8 = 204 \quad a_8 = ?$

8th row has 36 seats

5.) A display of shoe boxes has 45 boxes the fourth row and has 94 boxes in the eleventh row. How many boxes are in the first row?

Sequence $a_n = a_1 + d(n-1)$

$a_4 = 45 \quad a_{11} = 94 \quad a_1 = ?$

$d = ? \quad n = ?$ *NEED A SYSTEM OF EQUATIONS*.

$45 = a_1 + d(4-1)$

$94 = a_1 + d(11-1)$

24 boxes in 1st row

6.) A radio station wants to give away money every day in August for a total of $124,000. They want to increase each day's giveaway amount by $100 but still keep the same total amount. How much should they give away on the first day? Series (special)

$a_1 = ? \quad n = 31 \quad d = 100$

$S_{31} = 124{,}000$

Gives away $2500

7.) A theater has 18 seats in the third row and 54 seats in the twelfth row. The last row in the theater has 94 seats. How many seats are in the entire theater? Series

$a_3 = 18 \quad d = ? \quad S_n = ?$

$a_{12} = 54 \quad n = ?$

$a_n = 94$

Theater has 1144 seats.

8.) Gary wants to save money for a trip that costs $252. He gets paid every week at his job and starts the first week of saving with $12. He increases this amount every week by $4. How many weeks will it take Gary to save up for his trip? Series (special)

$S_n = \frac{n}{2}(2a_1 + d(n-1))$

$S_n = 252$

$a_1 = 12$

$d = 4$

$n = ?$

* will get a quadratic function solve need to solve using quadratic eqn or factoring.

9 weeks

Ex. 1

$a_{22} = 21{,}700 + 1{,}472(22-1)$

$a_{22} = 21{,}700 + 1{,}472(21)$

$a_{22} = 52{,}612$

Ex. 2

Step 1 Find a_{34}

$a_n = a_1 + d(n-1)$

$a_{34} = 20 + 2(34-1)$

$a_{34} = 86$

Step 2 Find S_{34}

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{34} = \frac{34}{2}(20 + 86)$

$S_{34} = 1802$

Ex. 3

$1850 = 12{,}500 + d(7-1)$

$-10{,}650 = 6d$

$d = -1775$

* use the word depreciates to indicate a negative value.

Ex. 4

$a_n = a_1 + d(n-1)$

$a_8 = 15 + 3(8-1)$

$a_8 = 36$

Use Elimination to solve Sys. of Eqn.

Ex. 5

$45 = a_1 + 3d$ → $-1(45 = a_1 + 3d)$ → $-45 = -a_1 - 3d$

$94 = a_1 + 10d$ → $94 = a_1 + 10d$

$49 = 7d$

$d = 7$

$45 = a_1 + 3(7)$

$45 = a_1 + 21$

$a_1 = 24$

Ex. 7

Step 1

$18 = a_1 + d(3-1)$ → $18 = a_1 + 2d$ → $-18 = -a_1 - 2d$

$54 = a_1 + d(12-1)$ → $54 = a_1 + 11d$ → $54 = a_1 + 11d$

$36 = 9d$

$d = 4$

Step 2 "a_1"

$18 = a_1 + 2(4)$

$18 = a_1 + 8$

$a_1 = 10$

Step 3 "n"

$94 = 10 + 4(n-1)$

$94 = 10 + 4n - 4$

$94 = 6 + 4n$

$4n = 88$

$n = 22$

Step 4 S_{22}

$S_{22} = \frac{22}{2}(10 + 94)$

$S_{22} = 1144$

Ex 6

$S_n = \frac{n}{2}\left(2a_1 + d(n-1)\right)$

$124{,}000 = \frac{31}{2}\left(2a_1 + 100(31-1)\right)$

$\frac{124{,}000}{15.5} = \frac{15.5}{15.5}\left(2a_1 + 3000\right)$

$8000 = 2a_1 + 3000$

$5000 = 2a_1$

$a_1 = 2500$

Ex. 8

$S_n = \frac{n}{2}\left(2a_1 + d(n-1)\right)$

$252 = \frac{n}{2}\left(2(12) + 4(n-1)\right)$

$504 = n(24 + 4n - 4)$

$504 = n(20 + 4n)$

$504 = 20n + 4n^2$

Quadratic; solve by method of your choice!

$4n^2 + 20n - 504 = 0$

$4(n^2 + 5n - 126) = 0$

Solved by factoring; use ZPP!

$4(n-9)(n+14) = 0$

$4 \neq 0$ ~~$\neq 0$~~ $n - 9 = 0$ $n + 14 = 0$

$n = 9$ $n \neq -14$

9 weeks