1.5 – Piecewise Functions

iecewise Function

Is a function that has its graph broken into different parts, thus its name "**Piece**"**wise.** The different functions are ***defined on VARIOUS DOMAIN VALUES*** (x-values). The x-values (independent) determine which function **piece** to use. (Inequalities after the word IF)

Step # 1 → Evaluate the function based on its domain (x) values ; start with #'s after the word IF.

Step # 2 → Make a table of values (points) ; indicate if those points are closed or open dots. ≤, ≥, = <, >, ≠

Step # 3 → After graphing (by hand – calculator is not helpful), CHECK for one MAJOR FEATURE: Piecewise function's graphs should NOT CROSS EACH OTHER!

Example 1: Evaluate each piecewise function. Validate points with the function's given graph.

Given Piecewise Function

a.) *Inequality tells you what #'s to use for "x".

$$f(x)=\begin{cases} x^2+2x-3 & \text{if } x<0 \\ x+1 & \text{if } x\geq 0 \end{cases}$$

Two pieces to the functions

Blue piece x^2+2x-3 "$x<0$" ← open dot at $(0,-3)$

Green piece → $x+1$ $x\geq 0$ ← closed dot at $(0,1)$

b.) *This function has four pieces! For parts to its graph.

$$f(x)=\begin{cases} -1 & \text{if } x<-3 \\ -|x|+5 & \text{if } -3\leq x<3 \cup x\neq 0 \\ -3 & \text{if } x=0 \\ 2x-10 & \text{if } x\geq 3 \end{cases}$$

Blue function → $y=-1$

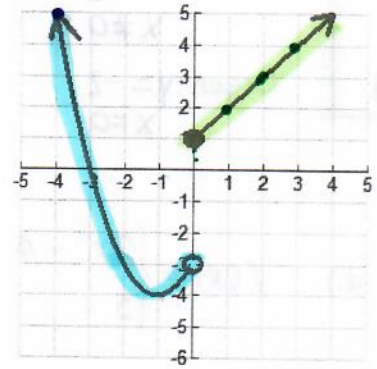
Green function → $y=-|x|+5$

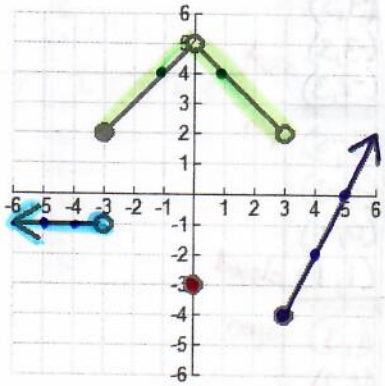
Red function → $y=-3$

Purple function → $y=2x-10$

$f(x)$ → "$y=$"

Graph of Piecewise Function





Evaluate/Complete Table of Values

$x<0$

x	Work to find f (x) or y	Pt (x,y)
−4	$(-4)^2+2(-4)-3 \rightarrow 5$	(−4, 5)
−2	$(-2)^2+2(-2)-3 \rightarrow -3$	(−2, −3)
−1	$(-1)^2+2(-1)-3 \rightarrow -4$	(−1, −4)
0	$(0)^2+2(0)-3 \rightarrow -3$	(0, −3) open dot!

$x\geq 0$

x	Work to find f (x) or y	Pt (x,y)
0	$(0)+1 \rightarrow 1$	(0, 1) closed dot
1	$(1)+1 \rightarrow 2$	(1, 2)
2	$(2)+1 \rightarrow 3$	(2, 3)
3	$(3)+1 \rightarrow 4$	(3, 4)

$x<-3$

x	Work to find f (x) or y	Pt (x,y)
-5	−1	(−5, −1)
-4	−1	(−4, −1)
-3	−1 *open dot*	(−3, −1)

$-3\leq x<3 \cup x\neq 0$

x	Work to find f (x) or y	Pt (x,y)
-3	$-|-3|+5 \rightarrow 2$ *closed*	(−3, 2)
-1	$-|-1|+5 \rightarrow 4$	(−1, 4)
0	$-|0|+5 \rightarrow 5$ *open*	(0, 5)
1	$-|1|+5 \rightarrow 4$	(1, 4)
3	$-|3|+5 \rightarrow 2$ *open*	(3, 2)

$x=0$

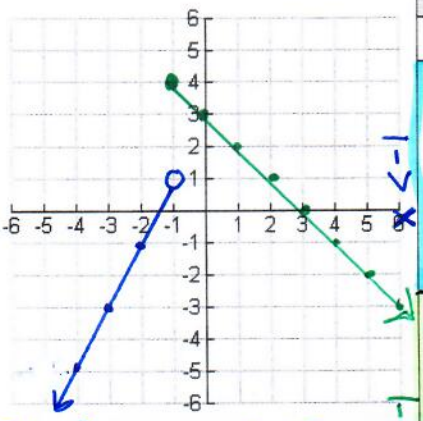
x	Work to find f (x) or y	Pt (x,y)
0	−3	(0, −3)

$x\geq 3$

x	Work to find f (x) or y	Pt (x,y)
3	$2(3)-10 \rightarrow -4$ *closed*	(3, −4)
4	$2(4)-10 \rightarrow -2$	(4, −2)
5	$2(5)-10 \rightarrow 0$	(5, 0)

Example 2: Make a table of domain values ("work") and graph each piecewise function.

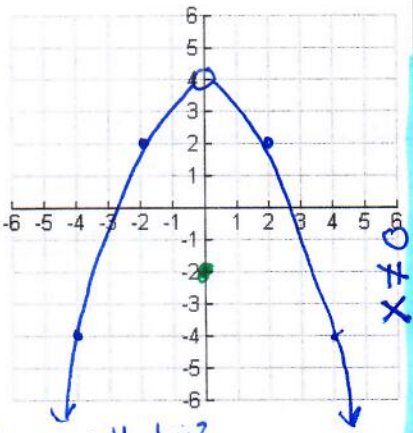
a.) $$f(x)=\begin{cases}2x+3 & \text{if } x<-1\\ 3-x & \text{if } x\geq -1\end{cases}$$



Blue function $y=2x+3$, $x<-1$

Table of Values		
x	Function	Pt (x,y)
-5	$2(-5)+3\to -7$	$(-5,-7)$
-4	$2(-4)+3\to -5$	$(-4,-5)$
-3	$2(-3)+3\to -3$	$(-3,-3)$
-2	$2(-2)+3\to -1$	$(-2,-1)$
-1	$2(-1)+3\to 1$	$(-1,1)$ open
-1	$3-(-1)$	$(-1,4)$ closed
0	$3-(0)$	$(0,3)$
1	$3-(1)$	$(1,2)$
2	$3-(2)$	$(2,1)$
3	$3-(3)$	$(3,0)$

b.) $$f(x)=\begin{cases}4-\frac{1}{2}x^2 & \text{if } x\neq 0\\ -2 & \text{if } x=0\end{cases}$$

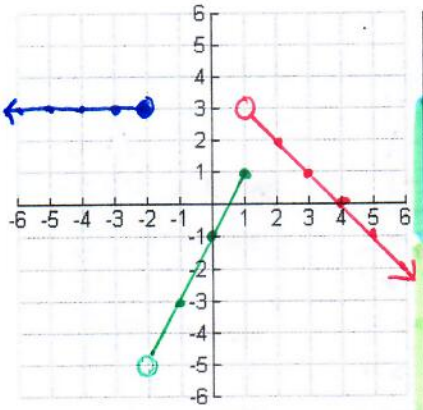


Blue $y=4-\frac{1}{2}x^2$, $x\neq 0$

Green $y=-2$, $x=0$

Table of Values		
x	function	y
-6	$4-\frac{1}{2}(-6)^2$	-14
-4	$4-\frac{1}{2}(-4)^2$	-4
-2	$4-\frac{1}{2}(-2)^2$	2
0	$4-\frac{1}{2}(0)^2$	4 open
2	$4-\frac{1}{2}(2)^2$	2
4	$4-\frac{1}{2}(4)^2$	-4
6	$4-\frac{1}{2}(6)^2$	-14
0	-2	-2 closed

c.) $$f(x)=\begin{cases}3 & \text{if } x\leq -2\\ 2x-1 & \text{if } -2<x\leq 1\\ 4-x & \text{if } x>1\end{cases}$$



Blue $y=3$, $x\leq -2$

Green $y=2x-1$, $-2<x\leq 1$

Red $y=4-x$, $x>1$

Table of Values		
x	function	Pt
-4	3	$(-4,3)$
-3	3	$(-3,3)$
-2	3	$(-2,3)$ closed
-2	$2(-2)-1$	$(-2,-5)$ open
-1	$2(-1)-1$	$(-1,-3)$
0	$2(0)-1$	$(0,-1)$
1	$2(1)-1$	$(1,1)$ closed
1	$4-(1)$	$(1,3)$ open
2	$4-(2)$	$(2,2)$
3	$4-(3)$	$(3,1)$
4	$4-(4)$	$(4,0)$

d.) $$f(x)=\begin{cases}2x^2-4 & \text{if } |x|\leq 2\\ 5 & \text{if } |x|>2\end{cases}$$

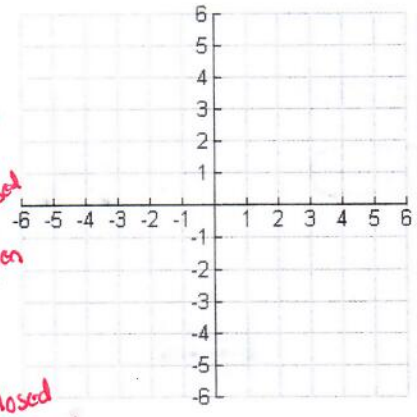


Table of Values

Example 3: Complete the problem.

During a particular year, the taxes owed by a married person filing separately with an adjusted gross income of x dollars is given by the piecewise function below:

$$T(x) = \begin{cases} 0.15x & \text{if } 0 \le x < 17{,}900 \\ 0.28(x-17{,}900)+2685 & \text{if } 17{,}900 \le x < 43{,}250 \\ 0.31(x-43{,}250)+9783 & \text{if } x \ge 43{,}250 \end{cases}$$

Find and interpret: T (70,000) + T (40,000)

$18,075.50 + $8873

The couples Adjusted gross income is $26,948.50

$T(x) = 0.31(x-43{,}250) + 9783$

$T(70{,}000) = 0.31(70{,}000-43{,}250)+9783$

$= \$18{,}075.50$

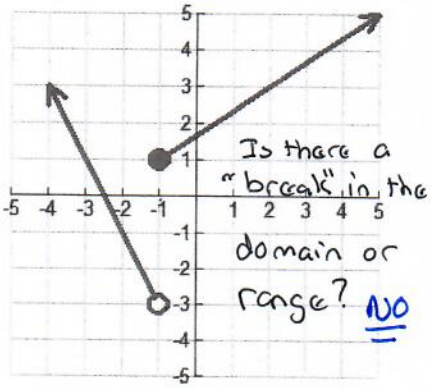
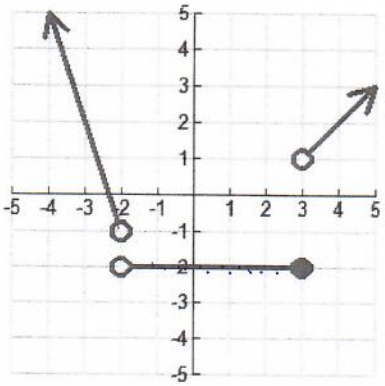
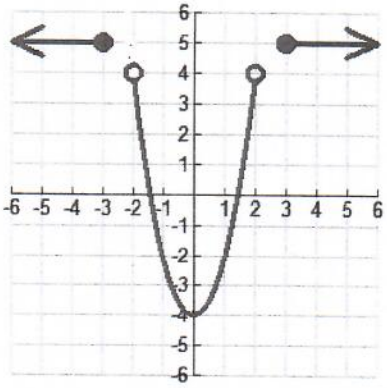
$T(x) = 0.28(x-17{,}900)+2685$

$T(40{,}000) = 0.28(40{,}000-17{,}900)+2685$

$= \$8873$

Example 4: Determine the domain and range of each piecewise graph in interval notation. [ ] ( )

Domain "x" Left to Right Range "y" Bottom to Top $-\infty$ ∞

Example 4a	Example 4b	Example 4c
		
D: $(-\infty, \infty)$ R: $(-3, \infty)$	D: $(-\infty,-2) \cup (-2, \infty)$ R: $[-2] \cup (-1, \infty)$	D: $(-\infty,-3] \cup (-2,2) \cup [3, \infty)$ R: $[-4,4) \cup [5]$