1.5 – Piecewise Functions

iecewise Function

Is a function that has its graph broken into different parts, thus its name "**Piece**"**wise.** The different functions are ***defined on VARIOUS DOMAIN VALUES*** (x-values). The x-values (independent) determine which function **piece** to use. (Inequalities after the word if)

Step # 1 → Evaluate the function based on its domain (x) values ; start with #'s after the word IF.

Step # 2 → Make a table of values (points) ; indicate if those points are closed or open dots. ≤, ≥, = <, >, ≠

Step # 3 → After graphing (by hand – calculator is not helpful), CHECK for one MAJOR FEATURE: Piecewise function's graphs should NOT CROSS EACH OTHER!

Example 1: Evaluate each piecewise function. Validate points with the function's given graph.

Given Piecewise Function

a.) *Inequality tells you what #'s to use for "x".

$$f(x)=\begin{cases} x^2+2x-3 & \text{if } x<0 \\ x+1 & \text{if } x\geq 0 \end{cases}$$

Two pieces to the functions

Blue piece x^2+2x-3 "$x<0$" ← open dot at $(0,-3)$

Green piece → $x+1$ $x\geq 0$ ← closed dot at $(0,1)$

b.) * This function has four pieces! For parts to its graph.

$$f(x)=\begin{cases} -1 & \text{if } x<-3 \\ -|x|+5 & \text{if } -3\leq x<3 \cup x\neq 0 \\ -3 & \text{if } x=0 \\ 2x-10 & \text{if } x\geq 3 \end{cases}$$

Blue function → $y=-1$

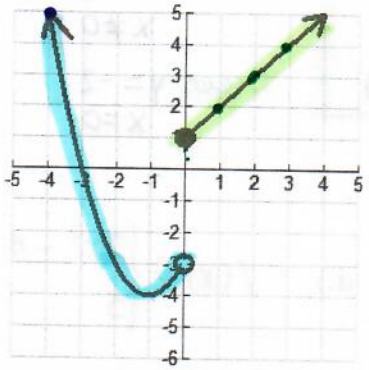
Green function → $y=-|x|+5$

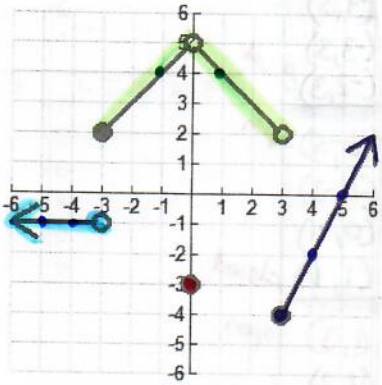
Red function → $y=-3$

Purple function → $y=2x-10$

$f(x)$ → "$y=$"

Graph of Piecewise Function





Evaluate/Complete Table of Values

x	Work to find f (x) or y	Pt (x,y)
−4	$(-4)^2+2(-4)-3 \rightarrow 5$	(−4, 5)
−2	$(-2)^2+2(-2)-3 \rightarrow -3$	(−2, −3)
−1	$(-1)^2+2(-1)-3 \rightarrow -4$	(−1, −4)
0	$(0)^2+2(0)-3 \rightarrow -3$	(0, −3) open dot!
0	$(0)+1 \rightarrow 1$	(0, 1) closed dot!
1	$(1)+1 \rightarrow 2$	(1, 2)
2	$(2)+1 \rightarrow 3$	(2, 3)
3	$(3)+1 \rightarrow 4$	(3, 4)

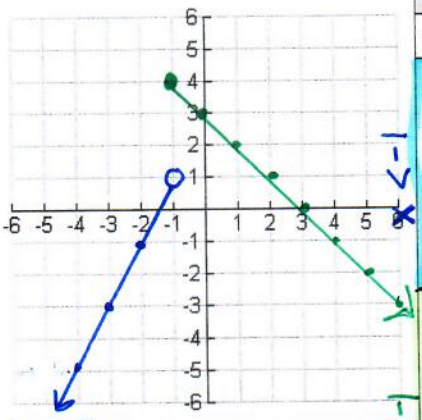
$x<0$ / $x\geq 0$

x	Work to find f (x) or y	Pt (x,y)
-5	−1	(−5, −1)
-4	−1	(−4, −1)
-3	−1 *open dot*	(−3, −1)
-3	$-|-3|+5 \rightarrow 2$ *closed*	(−3, 2)
-1	$-|-1|+5 \rightarrow 4$	(−1, 4)
0	$-|0|+5 \rightarrow 5$ *open*	(0, 5)
1	$-|1|+5 \rightarrow 4$	(1, 4)
3	$-|3|+5 \rightarrow 2$ *open*	(3, 2)
0	−3	(0, −3)
3	$2(3)-10 \rightarrow -4$ *closed*	(3, −4)
4	$2(4)-10 \rightarrow -2$	(4, −2)
5	$2(5)-10 \rightarrow 0$	(5, 0)

$x<-3$ / $-3\leq x<3 \cup x\neq 0$ / $x=0$ / $x\geq 3$

Example 2: **Make a table of domain values ("work") and graph each piecewise function.**

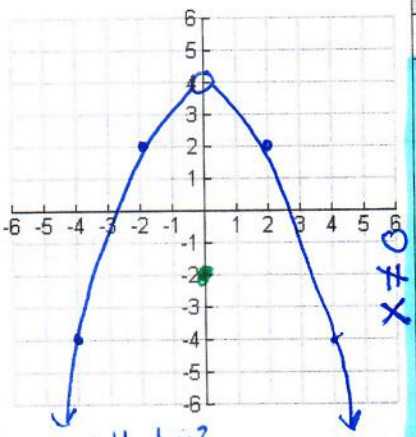
a.) $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \geq -1 \end{cases}$



Blue function $y = 2x+3$, $x < -1$

Table of Values		
x	Function	Pt (x,y)
-5	$2(-5)+3 \to -7$	(-5, -7)
-4	$2(-4)+3 \to -5$	(-4, -5)
-3	$2(-3)+3 \to -3$	(-3, -3)
-2	$2(-2)+3 \to -1$	(-2, -1)
-1	$2(-1)+3 \to 1$	(-1, 1) open
-1	$3-(-1)$	(-1, 4) closed
0	$3-(0)$	(0, 3)
1	$3-(1)$	(1, 2)
2	$3-(2)$	(2, 1)
3	$3-(3)$	(3, 0)

b.) $f(x) = \begin{cases} 4 - \frac{1}{2}x^2 & \text{if } x \neq 0 \\ -2 & \text{if } x = 0 \end{cases}$

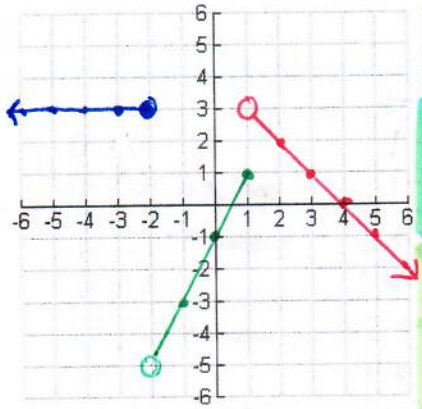


Blue $y = 4 - \frac{1}{2}x^2$, $x \neq 0$

Green $y = -2$, $x = 0$

Table of Values		
x	function	y
-6	$4 - \frac{1}{2}(-6)^2$	-14
-4	$4 - \frac{1}{2}(-4)^2$	-4
-2	$4 - \frac{1}{2}(-2)^2$	2
0	$4 - \frac{1}{2}(0)^2$	4 open
2	$4 - \frac{1}{2}(2)^2$	2
4	$4 - \frac{1}{2}(4)^2$	-4
6	$4 - \frac{1}{2}(6)^2$	-14
0	-2	-2 closed

c.) $f(x) = \begin{cases} 3 & \text{if } x \leq -2 \\ 2x-1 & \text{if } -2 < x \leq 1 \\ 4-x & \text{if } x > 1 \end{cases}$



Blue $y = 3$, $x \leq -2$

Green $y = 2x-1$, $-2 < x \leq 1$

Red $y = 4-x$, $x > 1$

Table of Values		
x	function	Pt
-4	3	(-4, 3)
-3	3	(-3, 3)
-2	3	(-2, 3) closed
-2	$2(-2)-1$	(-2, -5) open
-1	$2(-1)-1$	(-1, -3)
0	$2(0)-1$	(0, -1)
1	$2(1)-1$	(1, 1) closed
1	$4-(1)$	(1, 3) open
2	$4-(2)$	(2, 2)
3	$4-(3)$	(3, 1)
4	$4-(4)$	(4, 0)

d.) $f(x) = \begin{cases} 2x^2 - 4 & \text{if } |x| \leq 2 \\ 5 & \text{if } |x| > 2 \end{cases}$

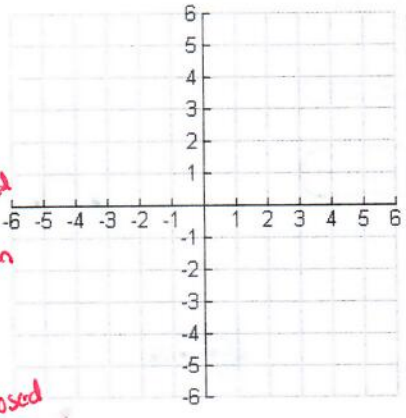


Table of Values	