1.2 – Arithmetic Series and Sigma Notation By Hand

Introduction to (General) Series

– **series** → the indicated Sum of terms in a sequence.

- distinguish the difference – sequence → 6 , 12 , 18 , … , 300 where it contains commas!

  series → 6 + 12 + 18 + … + 300 where it contains "+" signs (No Commas)

Tells you How many terms to add

- series notation – $S_n = \underbrace{a_1 + a_2 + a_3 + \dots + a_n}_{\text{sum of the 1st term thru the "n" term}}$

S_n: Sum of the 1st "n" terms

Example 1: Find the sum for each given sequence.

a.) Given: $a_n = 2n + 6$ ← Explicit Form Find: S_4 Step 1 Find 1st 4 terms $a_1 = 2(1) + 6 = 8$ $a_2 = 2(2) + 6 = 10$ $a_3 = 2(3) + 6 = 12$ $a_4 = 2(4) + 6 = 14$ Step 2 Find sum $S_4 = 8 + 10 + 12 + 14$ $S_4 = 44$	b.) Given: $a_n = 4(3n - 2)$ Find: S_3 Step 1 $a_1 = 4(3(1) - 2) = 4$ $a_2 = 4(3(2) - 2) = 16$ $a_3 = 4(3(3) - 2) = 28$ Step 2 $S_3 = 4 + 16 + 28$ $S_3 = 48$

Specific Series # 1 – Arithmetic Series

– **arithmetic series** → the indicated Sum of terms in an Arithmetic Sequence ($a_n = a_1 + d(n-1)$)

where it's represented by the following formula: $S_n = \frac{n}{2}(a_1 + a_n)$

S_n: Sum of the first "n" terms

a_1: 1st Term

a_n: "n" term (last term) $a_n = a_1 + d(n-1)$

Example 2: Find the sum of each arithmetic series.

a.) $a_1 = 8$, $a_n = 146$, $n = 24$ $S_n = \frac{n}{2}(a_1 + a_n)$ $S_{24} = \frac{24}{2}(8 + 146)$ $S_{24} = 1{,}848$	b.) $a_1 = 21$, $d = -3$, $n = 17$ Step 1 Find a_{17} $a_{17} = 21 + (-3)(17 - 1)$ $a_{17} = -27$ Step 2 Find S_{17} $S_{17} = \frac{17}{2}(21 - 27)$ $S_{17} = -51$	c.) 9 + 22 + 35 + … + 776 $a_1 = 9$ $d = 13$ $a_n = 776$ $n = ?$ Step 1 Find "n" $776 = 9 + 13(n - 1)$ $776 = 9 + 13n - 13$ $776 = 13n - 4$ $n = 60$ Step 2 Find S_{60} $S_{60} = \frac{60}{2}(9 + 776)$ $S_{60} = 23{,}550$

$S_n = \frac{n}{2}(a_1 + a_n)$ → Series

Example 3: Use the arithmetic series formula to complete each problem. $a_n = a_1 + d(n-1)$ → Sequence

a.) How many terms were added to get a sum of 5,958 when the first term is 8 and the last term in the series is 323? $a_1 = 8$ $a_n = 323$ $S_n = 5958$ $n = ?$ $5958 = \frac{n}{2}(8 + 323)$ $(2)5958 = \frac{n}{2}(331)(2)$ $11916 = 331n$ $n = 36$ 36 terms	b.) What is the second term of an arithmetic series if the first term is 9, the nth term is 105, and the sum is 741? $a_1 = 9$ $a_n = 105$ $S_n = 741$ $n = ?$ $d = ?$ $a_2 = ?$ Step 1 Find "n" $741 = \frac{n}{2}(9 + 105)$ $741 = \frac{n}{2}(114)$ $1482 = 114n$ $n = 13$ Step 2 Find "d" $a_{13} = 105$ $a_1 = 9$ $n = 13$ $105 = 9 + d(13-1)$ $105 = 9 + 12d$ $d = 8$ Step 3 find a_2 $a_2 = 9 + 8 = 17$

Sigma Notation (By Hand Method)

– **sigma notation** → a less lengthy and more concise way to write out a series.

- The following is a simple representation of Sigma Notation:

Tells what n^{th} term to stop. Explicit Formula. Tells what "n" to start.

$$\sum_{n=1}^{4} 3n = 3(1) + 3(2) + 3(3) + 3(4) = 3 + 6 + 9 + 12 \Rightarrow S_4 = 30$$

$n=1$ $n=2$ $n=3$ $n=4$

Example 4: Find the sum.

a.) $\sum_{n=3}^{5} 4n + 3$ Step 1 $n=3 \to 4(3)+3 = 15$ $n=4 \to 4(4)+3 = 19$ $n=5 \to 4(5)+3 = 23$ Step 2 $S_3 = 15 + 19 + 23$ $S_3 = 57$	b.) $\sum_{n=1}^{5} 2n - 4$ Step 1 $2(1) - 4 = -2$ $2(2) - 4 = 0$ $2(3) - 4 = 2$ $2(4) - 4 = 4$ $2(5) - 4 = 6$ $S_5 = -2 + 0 + 2 + 4 + 6$ $S_5 = 10$	c.) $\sum_{n=5}^{26} 5 - 3n$ $S_n = \frac{n}{2}(a_1 + a_n)$ Step 1 $a_5 = 5 - 3(5) = -10 \to a_1$ $a_{26} = 5 - 3(26) = -73 \to a_n$ Step 2 $n = 26 - 5 + 1$ $n = 22$ with sigma notation always add 1 when finding "n" Step 3 Use S_n Formula $S_{22} = \frac{22}{2}(-10 - 73)$ $S_{22} = -913$