


# 1.1 – Arithmetic Sequences

## Introduction to (General) Sequences

– sequence → a list of a pattern in a certain order  
 where each part of the pattern is called a Term.

• sequence notation –  $a_1, a_2, a_3, \dots, a_n$   
 $a_1$  is read as "a of 1"  
 1<sup>st</sup> Term, 2<sup>nd</sup> Term, 3<sup>rd</sup> Term, "n<sup>th</sup> term"  
 \* "n" is always a positive whole #!  
 is a function of the sequence.

• sequence representations – we can represent a sequence in three ways:

- Pictures → Ex:  {Add stem CW and color last circle.}
- Words → Ex: applying, bathing, cuddling, driving, eating {ABC order and verbs ending in "ing"}
- Numbers → Ex: 3, 10, 24, 45, 73, 108 {Adding multiples of 7  
 $+7, +7, +7, +7, +7, +7$   
 $+7, +14, +21, +28, +35$ }

## Specific Sequence # 1 – Arithmetic Sequence

– arithmetic sequence → a sequence where the difference between any two consecutive terms is a constant, called "d", the common difference.  
 \* Adding a Fixed # to the previous term to get to the next term.

### Example 1: Complete each problem.

<p>a.) Is the given sequence arithmetic?                  -2, 4, 10, 16, ...                  If so, what is the value of d?</p> <p><math>\frac{16}{6} - \frac{10}{6} = \frac{6}{6} = 1</math>  <math>\frac{10}{6} - \frac{4}{6} = \frac{6}{6} = 1</math>  <math>\frac{4}{6} - \frac{-2}{6} = \frac{6}{6} = 1</math> ← common difference</p> <p>Yes; <math>d = 6</math></p>	<p>b.) Is the given sequence arithmetic?                  18, 15, 12, 10, ...                  If so, what is the value of d?</p> <p><math>\frac{12}{-2} - \frac{15}{-3} = \frac{-12}{-2} - \frac{-15}{-3} = 6 - 5 = 1</math>  <math>\frac{15}{-3} - \frac{18}{-2} = \frac{-15}{-3} - \frac{-18}{-2} = 5 - 9 = -4</math></p> <p>NO                  Not the same!</p>	<p>c.) An arithmetic sequence has <math>a_1 = 20</math> and <math>d = -4</math>. What is the fifth term of the sequence?</p> <p><math>a_1 = 20</math>  <math>a_2 = 20 + (-4) = 16</math>  <math>a_3 = 16 + (-4) = 12</math>  <math>a_4 = 12 + (-4) = 8</math>  <math>a_5 = 8 + (-4) = 4</math></p> <p><math>a_5 = 4</math></p>
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## "Nth Term Formula" of Arithmetic Sequence: Used to find ANY term of an arithmetic sequence

Consider an arithmetic sequence whose first term is " $a_1$ " and whose common difference is " $d$ ":

$a_1$	→	1st term ( $a_1$ )
$a_1 + d$	→	2nd term ( $a_2 = a_1 + d$ )
$a_1 + d + d = a_1 + 2d$	→	3rd term ( $a_3 = a_1 + 2d$ )
$a_1 + 2d + d = a_1 + 3d$	→	4th term ( $a_4 = a_1 + 3d$ )
$a_1 + 3d + d = a_1 + 4d$	→	5th term ( $a_5 = a_1 + 4d$ )

(General) n<sup>th</sup> term Formula:  $a_n = a_1 + d(n-1)$  or  $a_n = a_1 + (n-1)d$  → some notes about this formula...

- formula will always be a linear equation (straight line)
- make sure your final nth term formula is SIMPLIFIED COMPLETELY

Perform all operations you can do!  
 (distribute and combine like terms)

**Example 2:** Find what is indicated for each arithmetic sequence.

*n<sup>th</sup> term* →  $a_n = a_1 + d(n-1)$  *Common difference* *the term you are looking for!*

<p>a.) <math>a_1 = -6</math> and <math>d = 7</math>, find the 16<sup>th</sup> term <math>n=16</math></p> $a_{16} = -6 + 7(16-1)$ $a_{16} = -6 + 7(15)$ <p style="text-align: center;"><u><math>a_{16} = 99</math></u></p>	<p>b.) Find <math>a_{40}</math> for the sequence <math>-9, -17, -25, \dots</math></p> $a_1 = -9 \quad d = -8$ $n = 40$ $a_{40} = -9 + (-8)(40-1)$ $a_{40} = -9 + (-8)(39)$ <p style="text-align: center;"><u><math>a_{40} = -321</math></u></p>	<p>c.) Write the nth term formula (equation) for the sequence <math>8, 17, 26, 35, \dots</math></p> $a_1 = 8 \quad a_n = 8 + 9(n-1)$ $d = 9 \quad a_n = 8 + a_n - 8$ $n = n \quad \underline{a_n = a_n - 1}$
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$\frac{35}{9} \frac{26}{9} \frac{17}{9}$   
 $\frac{-26}{9} \frac{-17}{9} \frac{-8}{9}$   
No r Simplified

**Example 3:** Considering all given sequences are arithmetic – Find what is asked.

<p>a.) The 20<sup>th</sup> term of the sequence is 101 and the common difference is 3. What is the first term?</p> $a_n = a_1 + d(n-1) \quad a_1 = ? \quad d = 3 \quad n = 20$ $a_{20} = 101$ $101 = a_1 + 3(20-1)$ $101 = a_1 + 3(19)$ $101 = a_1 + 57$ <p style="text-align: center;"><u><math>a_1 = 44</math></u></p>	<p>b.) What is the common difference for the sequence where the first term is 13 and the 28<sup>th</sup> term is <math>-149</math>?</p> $-149 = 13 + d(28-1)$ $-149 = 13 + 27d$ $-162 = 27d$ <p style="text-align: center;"><u><math>d = -6</math></u></p>
<p>c.) Which term of the sequence <math>1, 5, 9, \dots</math> is 97? <math>a_1 = 1 \quad d = 4 \quad a_n = 97 \quad n = ?</math></p> $97 = 1 + 4(n-1)$ $97 = 1 + 4n - 4$ $97 = 4n - 3$ $100 = 4n$ $n = 25$ <p style="text-align: center;"><u>25<sup>th</sup> term</u></p>	<p>d.) The thirty-second term in the sequence is 534 and the fourteenth term is 228. What is the tenth term of the sequence? *Set up a sys. of Eq. to help solve for <math>a_1</math> and <math>d</math>.</p> $a_{32} = 534$ $a_{14} = 228$ $a_{10} = ?$ $a_n = a_1 + d(n-1)$ $534 = a_1 + d(32-1)$ $228 = a_1 + d(14-1)$ $534 = a_1 + 31d$ $228 = a_1 + 13d$ $306 = 18d \rightarrow d = 17$ $534 = a_1 + (17)(32-1)$ $534 = a_1 + 527$ $a_1 = 7$ $a_{10} = 7 + (17)(10-1)$ <p style="text-align: center;"><u><math>a_{10} = 160</math></u></p>

– **arithmetic means** → represent the terms between any two nonsuccessive of an arithmetic sequence

Ex: Circle the 3 arithmetic means between 30 and 74:  $19, 30, 41, 52, 63, 74, 85, 96, \dots$

**Example 4:** Complete each problem. Assume both sequences are arithmetic.

<p>a.) Find the four arithmetic means for <math>16, 31, 46, 61, 76, 91</math></p> $a_n = a_1 + d(n-1) \quad a_1 = 16 \quad a_6 = 91$ $a_1 = 16 + d(6-1)$ $91 = 16 + 5d$ $d = 15$ $a_2 = 16 + 15 = 31$ $a_3 = 31 + 15 = 46$ $a_4 = 46 + 15 = 61$ $a_5 = 61 + 15 = 76$	<p>b.) Find the <u>two arithmetic means</u> between 52 and 10.</p> $52, 38, 24, 10$ $10 = 52 + d(4-1)$ $-42 = 3d$ $d = -14$ $a_2 = 52 + (-14)$ $a_2 = 38$ $a_3 = 38 + (-14)$ $a_3 = 24$
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