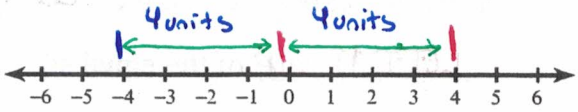
1.3 – Absolute Value Equations

Absolute Value (of a number) → the DISTANCE from ZERO on a number line

- If a is any real number, then $|a| = a$ and $|-a| = a$
- For example, $|4| =$ 4 and $|-4| =$ 4 because



- The absolute value (distance/length) will always be a POSITIVE number.

Absolute Value Equation → If $|x| = b$, then $x = b$ and $x = -b$

- Make sure the Absolute Value is ISOLATED before separating them into **TWO EQUATIONS**.
- Make sure to CHECK solutions since there are possibilities that some MAY or MAY NOT make the original absolute value equation TRUE. These solutions that DO NOT WORK are called Extraneous Solutions.

* If absolute value equation is equal to a NEGATIVE # after isolation then answer is $\emptyset$ (No solution)
- There is NO Distributing over absolute value bars!!!! ie $2|x+2|$ * Can not climb over the wall!

Example 1: Solve each absolute equation algebraically. * Make sure to CHECK your solutions *

a.) $|3x+6| = 18$ The abs. val is isolated. Make 2 eqns.

$3x+6=18$ | $3x+6=-18$
$-6 \quad -6$ | $-6 \quad -6$
$\frac{3x}{3} = \frac{12}{3}$ | $\frac{3x}{3} = \frac{-24}{3}$
$x = 4$ ✓ | $x = -8$ ✓

Check your solutions

set notation → $\{-8, 4\}$

b.) $5 - 3|2x-10| = -43$ * Need to isolate abs val before 2 eqns.

$-5 \quad -5$
$\frac{-3|2x-10|}{-3} = \frac{-48}{-3}$

$|2x-10| = 16$

$2x-10=16$ | $2x-10=-16$
$+10 \quad +10$ | $+10 \quad +10$
$\frac{2x}{2} = \frac{26}{2}$ | $\frac{2x}{2} = \frac{-6}{2}$
$x = 13$ ✓ | $x = -3$ ✓

$\{-3, 13\}$

c.) $4|5-2x| - 3 = -79$

$+3 \quad +3$
$\frac{4|5-2x|}{4} = \frac{-76}{4}$

$|5-2x| = -19$

isolated = negative #

$\emptyset$

d.) $6 + 7|3x+9| = 6$

$-6 \quad -6$
$\frac{7|3x+9|}{7} = \frac{0}{7}$

$|3x+9| = 0$

$3x+9=0$
$-9 \quad -9$
$\frac{3x}{3} = \frac{-9}{3}$
$x = -3$ ✓

$\{-3\}$

e.) $|x+6| = 3x-2$

$x+6=3x-2$ | $x+6=-(3x-2)$
$-x \quad -x$ | $x+6=-3x+2$
$6=2x-2$ | $+3x \quad +3x$
$+2 \quad +2$ | $4x+6=2$
$\frac{8}{2} = \frac{2x}{2}$ | $-6 \quad -6$
$x = 4$ ✓ | $\frac{4x}{4} = \frac{-4}{4}$
| $x = -1$ ✗

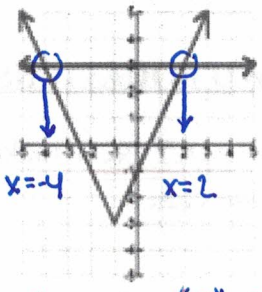
$\{4\}$

Extraneous Solution

Absolute Value equations can also be solved Graphically.

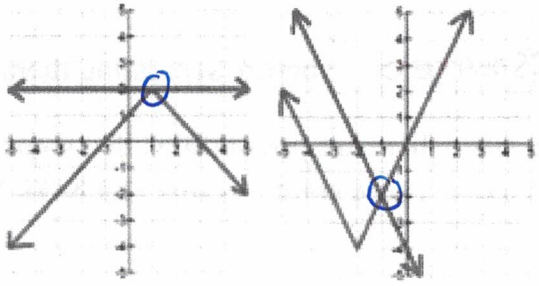
- Treat like a System of Equations.
- Input the expression on the Left side of equal sign (=) into Y_1. Input the expression on the RIGHT SIDE of the equal sign (=) into Y_2.
- Use **2nd** **Trace** **#5:intersect** **Enter** (x3) to find solutions; only want the X-values.

Types of Solutions Produced by Absolute Value Equations "GRAPHICALLY"

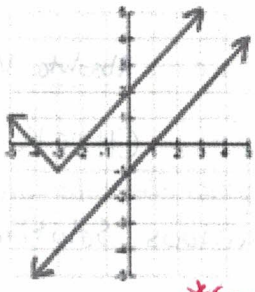


* only want "x" *

# of Solutions: 2



# of Solutions: 1



*Graphs do not intersect

# of Solutions: 0; NO solutions

Example 2: Solve each absolute equation graphically. **Remember only want the "x-value(s)"**

a.) $2-|x-1|=2$ (Y_1, Y_2) {1}

b.) $|x+3|-1=x-1$ (Y_1, Y_2) ∅

c.) $-|x-2|+4=\frac{1}{2}x+1$ (Y_1, Y_2) {-2, 3.3}

Example 3: Solve each absolute value word problem.

a.) A machine fills Quaker Oatmeal containers with 32 ounces of oatmeal. After the containers are filled, another machine weighs them. If the container's weight differs from the desired weight by more than 0.5 ounces, the container is rejected. What are the heaviest and lightest acceptable weights for the Quaker Oatmeal container?

32 oz is desired tolerance of ± .5 oz

x = weight of container

$|x-32|=.5$ → $x-32=.5 \rightarrow 32.5$; $x-32=-.5 \rightarrow 31.5$

The heaviest container can be 32.5 oz and the lightest container can be 31.5 oz.

b.) Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. What is the minimum brewing temperature for an excellent cup of coffee?

$|x-200|=5$

$x-200=5$, $x=205°$

$x-200=-5$, $x=195°$ ↑minimum

The minimum brewing temperature is 195°.